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# CONTROL OF MATHEMATICAL MODELING PROCESS OF DYNAMICS OF HARMFUL SUBSTANCES CONCENTRATIONS ON THE BASIS OF ONTOLOGICAL APPROACH

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**Abstract:** The problem of building a mathematical model of the dynamics of nitrogen dioxide concentrations at different parts of the city is considered in the paper. The peculiarities of the construction of such models on the basis of periodic measurement of concentrations of harmful substances and identification on the basis of the measurements obtained are considered.

This paper also proposes an ontological approach as a control tool that greatly simplifies the systematic standardized methods of the models storage, the process of their construction and appropriate usage in practice.

The use of the ontological model allows formalizing the process of obtaining, storing and using relevant knowledge and is suitable for more intelligent systems, such as identification of obviously erroneous solutions based on the model, predictive control of the model, optimization of the decision-making process based on knowledge and modeling of an appropriate technological flow chart.

This paper also describes the features of the construction of the corresponding ontological model, the pattern of choice of a nonlinear model with "switching" to different conditions. Relevant experimental studies have also been conducted to confirm the effectiveness of the proposed approach.

**Key words:** mathematical modeling; interval analysis; knowledge management; ontological approach.

## 1. Introduction

Processes of spreading harmful emissions into the atmosphere are the subject of a number of works. The most considerable sources of pollution in cities are vehicles. This type of pollution sources in large cities accounts for up to 80 % of all pollution. Being in the surface layer of the atmosphere in the gaseous state, pollution accumulates and, therefore, pollutes soils and water resources. One of the most harmful pollutants contained in high concentrations in the exhaust gases of vehicles is nitrogen dioxide which, when mixed with water, converts into nitric acid and nitrates in soils and groundwater.

To reduce the negative impact of this pollutant on the environment, it is necessary to develop a mechanism for monitoring and regulating pollution processes. Mathematical models of the dynamics of nitrogen dioxide concentrations in different parts of the city can be a tool for such regulation. Such models can be built on the basis of periodic measurements of concentrations of this substance and identification on the basis of the obtained measurements. Due to the fact that the measured concentrations contain measurement errors and the number of measurements is limited, the paper proposes using the interval data analysis to build such models. The most complicated problem is to build adequate models for different conditions which could be, for example, different measurement points in the city or the weather conditions. It means, that if the model is built under certain conditions, it may be unsuitable for other conditions. Therefore, to take into account a wide range of conditions, in the paper it is proposed to use nonlinear models with "switches" to different conditions. As it is known, the dynamics of processes is described by differential equations, or their discrete analogues in the form of difference equations. This class of models is used in the construction of models of the dynamics of nitrogen dioxide concentrations due to pollution of the surface layer of the atmosphere by harmful emissions from vehicles.

It should be noted that such models are built in the form of algebraic equations or difference equations. The use of algebraic equations in this case causes certain difficulties which are associated with the difficulty of adjusting to different conditions. Recently, interval difference equations have been used in this area. Adjusting them to a specific object of study is easier, because in the process of building the model of the dynamics of nitrogen dioxide concentrations in different areas it is easier to adjust a difference scheme. Very often such models are inaccurate and, accordingly, the technology itself becomes inefficient.

Therefore, it was decided to improve the existing technology through the use of an ontological model that contains all obtained knowledge about monitoring and regulating the processes of pollution by harmful substances.

This work is devoted to solving this issue with the use of ontology and appropriate means of its implementation to meet these needs.

#### 2. Statement of the research problem

Differential equations, or their differential analogues, are often used to model the processes of pollution of the surface layer of the atmosphere by harmful emissions from non-stationary sources of pollution, such as motor vehicles. For the structural identification of difference equations, we can use a method based on behavioral models of the bee colony [1].

Now let us consider the problem of parametric identification of interval models of dynamic objects. First, let us make assumptions about the structure of the mathematical model.

So, the mathematical model of the object can be considered in the form of the following difference equation:

$$v_{k} = \vec{f}^{T}(v_{k-d}, ..., v_{k-1}, \vec{u}_{0}, .., \vec{u}_{d-1}, \vec{u}_{k}) \cdot \vec{g}, k = d, ..., K$$
(1)

where  $v_k$  is simulated concentration of nitrogen dioxide on the time discrete  $k = d, ..., K; \vec{u}_0, ..., \vec{u}_k$  are vectors of input variables that specify the conditions of construction and conditions of application of the model; d is the order of discrete dynamic model;  $\vec{g}$  is a vector of unknown model parameters;  $\vec{f}^T(\bullet)$  is a vector of known basic functions.

Expression (1) is used to present the linear difference equation according to the parameters. In general, without violating the generality of the consideration, we can assume that the equation can be nonlinear.

The results of measurements of nitrogen dioxide concentrations are presented in the form of numerical intervals of their possible values at a certain point and in uniformly recorded time discrete:

$$[z_k^-; z_k^+], k = 0, ..., K,$$
(2)

where  $z_k^-$ ,  $z_k^+$  are the lower and upper limits of the range of possible values of the measured concentrations respectively, in time discrete k = 0, ..., K.

Compliance with the requirements for ensuring the accuracy of the mathematical model within the accuracy of interval data leads to the following interval system:

$$\begin{cases} [\hat{v}_{0}^{-}; \hat{v}_{0}^{+}] \subseteq [z_{0}^{-}; z_{0}^{+}], ..., \\ .... [\hat{v}_{d-1}^{-}; \hat{v}_{d-1}^{+}] \subseteq [z_{d-1}^{-}; z_{d-1}^{+}]; \\ z_{k}^{-} \leq \vec{f}^{T} ([\hat{v}_{k-d}], ..., [\hat{v}_{k-1}], \vec{u}_{0}, ..., , \\ ..., \vec{u}_{k}) \cdot \hat{\vec{g}} \leq z_{k}^{+}, k = d, ..., K. \end{cases}$$

$$(3)$$

in which the first line denotes the initial conditions, and the next ones – the conditions of agreement of experimental data with predicted figures which will be calculated on the basis of a mathematical model whose parameters must be calculated from system (3). Based on the general representation of the object model in the form of difference equation (1), the specified mathematical model, when the values of the parameters have already been calculated, is given by the following expression:

$$[\hat{v}_{k}] = [\hat{v}_{k}^{-}; \hat{v}_{k}^{+}] = \vec{f}^{T} ([\hat{v}_{k-d}], ..., ...[\hat{v}_{k-1}], \vec{u}_{0}, ..., \vec{u}_{d-1}, \vec{u}_{k}) \cdot \hat{\vec{g}}, \ k = d, ..., K.$$

$$(4)$$

where  $[\hat{v}_k]$  is the interval estimation of nitrogen dioxide concentration on time discrete k = d, ..., K;  $\hat{\vec{g}}$  is a vector of values of model parameters. All calculations, both in the construction of model (4) and in its application are performed using interval arithmetic. Therefore, such models are called interval discrete models of dynamic objects (IDMDO).

As we can see, in the case of parametric identification of interval models of nitrogen dioxide dynamics, we deal with a mathematical problem, the essence of which is to completely solve or find at least one solution of the interval system of nonlinear algebraic equations (ISNAE).

Compatibility of ISNAE (3) means the range of values of the predicted concentration of nitrogen dioxide  $[\hat{v}_k]$  on temporal discrete k = 0, ..., K to intervals  $[z_k^-; z_k^+]$ , obtained experimentally in the same discrete, i.e., when the following conditions are met:

$$[\hat{v}_k] \subset [z_k^-; z_k^+], k = 0, ..., K.$$
(5)

Therefore, the iterative procedure for the method of estimating the solution of ISNAE (3) should be based on the "quality" of the estimation of the mathematical model parameters represented by the difference equation (4) investigated at each iteration. The quality of parameter estimation for each l-th iteration is determined by the value  $\delta(\hat{\vec{g}}_{i})$  as the difference of the centers the most distant from each other, namely, those modeled on the basis of the difference scheme (4) and the experimental intervals  $[z_k^-; z_k^+]$  for each discrete in the case when these intervals do not intersect.

In the case of intersection of these intervals, the function  $\delta(\hat{\vec{g}}_{_{l}})$  will be determined by the smallest section width among the predicted and experimental intervals. An expression for function  $\delta(\hat{\vec{g}}_{_{l}})$ , for both above cases, will be presented as follows, respectively:

$$\delta(\hat{\overline{g}}) = \max_{i=1,\dots,N} \left\{ \left| mid([\hat{\nu}_k]) - mid([z_k^-; z_k^+]) \right| \right\}$$
  
if  $[\hat{\nu}_k] \cap [z_k^-; z_k^+] = \emptyset, \exists k = 0, \dots, K,$  (6)

$$\delta(\hat{\vec{g}}) = \max_{i=1,\dots,N} \left\{ wid([\hat{v}_k]) - wid([\hat{v}_k] \cap [z_k^-; z_k^+]) \right\}$$
  
if  $[\hat{v}_k] \cap [z_k^-; z_k^+] \neq \emptyset, \forall k = 0,\dots,K.$  (7)

It should be noted that ISNAE (3) is complex because each equation is formed recurrently. Therefore, in the case of the development of IDMDO, the task will be to calculate only one solution of ISNAE (3).

It should be also noted that conditions (5) for the completion of the iterative procedure, in turn, ensure the fulfillment of the following condition:

$$\delta(\vec{g}_{\mu}) = 0. \tag{8}$$

To use formulas (6) and (7) in computational procedures to calculate the values of the function  $\delta(\hat{\vec{g}}_{_{I}})$ , let us rewrite them in an expanded form:

(1)

$$\delta(\hat{g}_{1}) = \max_{i=1,...,N} \left\{ \begin{array}{l} mid(\vec{f}^{T}([\hat{v}_{k-d}],...,\\....[\hat{v}_{k-1}], \vec{u}_{0},...,\vec{u}_{d-1},\vec{u}_{k}) \cdot \hat{\vec{g}}) - \\ -mid([z_{k}^{-};z_{k}^{+}]) \right\} \right.$$
(9)  
if  $[\hat{v}_{k}] \cap [z_{k}^{-};z_{k}^{+}] = \emptyset, \exists k = 0,...,K,$ 

$$\delta([\hat{\vec{g}}_{i}^{-};\hat{\vec{g}}_{i}^{+}]) = \max_{i=1,...,N} \begin{cases} wid(\vec{f}^{T}([\hat{v}_{k-d}],..., [\hat{v}_{k-1}], \vec{u}_{0},...,\vec{u}_{d-1},\vec{u}_{k}) \cdot \hat{\vec{g}}) - (1-i) \\ \dots - ($$

$$-wid(\vec{f}^{T}([\vec{v}_{k-d}],...,[\vec{v}_{k-1}],\vec{u}_{0},...,])$$

$$...\vec{u}_{d-1},\vec{u}_{k})\cdot\hat{\vec{g}})\cap [z_{k}^{-};z_{k}^{+}]$$

$$(10)$$

if 
$$[\hat{v}_k] \cap [z_k^-; z_k^+] \neq \emptyset, \forall k = 0, ..., K$$
.

However, an important question how to optimally organize the iterative procedure for calculating the sequence of interval estimates of the components of the parameter vectors  $\hat{\vec{g}}_1, \hat{\vec{g}}_2, ..., \hat{\vec{g}}_l$ ,... remains unanswered. Obviously, to ensure the convergence of the iterative procedure, it is necessary to ensure such a sequence of the estimation of the values of the function  $\delta(\hat{\vec{g}}_1), \delta(\hat{\vec{g}}_2), ..., \delta(\hat{\vec{g}}_l), ...$  calculated by expression (9), or (10) which can lead to fulfilment of following conditions:

$$\delta(\hat{\vec{g}}_1) > \delta(\hat{\vec{g}}_2) >, \dots, > \delta(\hat{\vec{g}}_l), \dots, > \delta(\hat{\vec{g}}_{l=L} \subset \Omega),$$
(11)

where  $\Omega$  in this case denotes the area of solutions of ISNAE (12).

As a result, the task of parametric identification of interval models of a dynamic object can be formulated in the form of an optimization problem:

$$\delta(\hat{\vec{g}}_l) \xrightarrow{\vec{g}_l} \min, \, \hat{g}_{jl} \in [g_{jl}^{low}; g_{jl}^{up}], \qquad (12)$$
  
$$j = 1, \dots, m, \, l = 1, \dots, S$$

where the value of the goal function  $\delta(\hat{g}_l)$  is calculated by formula (9), or (10).

## 3. Solution method

Basing on the results of [нема посилання], we use the principles of behavioral models of bee colonies to build a method of parametric identification.

*Initialization phase.* Vectors determining the possible minimum points of the goal function from (12), that is, the vectors of parameter estimates are denoted by  $\hat{\vec{g}}_l$ . In the context of the behavioral model of the bee colony, this means that each vector of coordinates of a nectar source corresponds to one l-th bee that studies it, and the number of the entire population is equal to S. In this phase

$$\hat{g}_{jl} = g_{jl}^{low} + rand(0,1) * (g_{jl}^{up} - g_{jl}^{low}), \quad (13)$$
$$j = 1, ..., m, l = 1, ..., S$$

where  $g_{jl}^{low}$ ,  $g_{jl}^{low}$  denote lower and upper limit of parameter values at the initialization stage.

Let us note that for the case of finding point estimates of parameters, contrary to the case of identification of interval estimates of parameters of the static object model, the interval  $[g_{jl}^{low}]$  degenerates to some numerical value  $g_{jl}^{low}$ .

It is worth noting that in this phase we also configure all the parameters of the algorithm.

## Phase of worker bees

In the context of the optimization problem, the phase of worker bees means the search for new estimates of solutions with smaller values of the objective function (9) or (10). To calculate the possible points of the local minimum of the goal function, we use the following formulas:

$$\widehat{g}_{jl}^{mcn} = \widehat{g}_{jl} + \Phi_{jl} * (\widehat{g}_{jl} - \widehat{g}_{jp}), 
j = 1,...,m, p \neq l = 1,...,S$$
(14)

After calculating the coordinates of possible minimum points  $\vec{g}_l^{mcn}$ , a pairwise comparison of existing and current values of parameter estimates using the objective function (9) or (10) is conducted:

$$\widehat{\tilde{g}}_{l} = \left\{ \widehat{\tilde{g}}_{l}, \text{ if } \delta(\widehat{\tilde{g}}_{l}) \leq \delta(\left(\widehat{\tilde{g}}_{l}^{mcn}\right) \right\} \\
\text{or } \widehat{\tilde{g}}_{l} = \left\{ \widehat{\tilde{g}}_{l}^{mcn}, \text{ if } \delta(\widehat{\tilde{g}}_{l}) > \delta\left(\widehat{\tilde{g}}_{l}^{mcn}\right) \right\}.$$
(15)

**Phase of bees** – **researchers**. In the context of the optimization problem, at this stage we determine the most probable points (vectors of parameter values), around which it is necessary to conduct a detailed study of the goal function. These points claim to provide local minima of the goal function. For these purposes, we use the probabilistic approach, namely, calculate the probabilities of the feasibility of the investigation of each point (given by the vector of parameter values) from those previously determined. The expression for calculating the specified probability is as follows:

$$P_l = \frac{1 - \delta(\vec{g}_l)}{\sum\limits_{l=1}^{S} (1 - \delta(\vec{g}_l))}.$$
(16)

It should be noted that in the case of significant discrepancies between the values of the goal function  $\delta(\hat{\vec{g}}_l)$  calculated for different points (vectors of parameter values), it is necessary to rewrite formula (16), taking into account the normalization of the values of this function. In this case, formula (16) takes the following form:

$$P_l = \frac{1}{\delta(\hat{\vec{g}}_l) \sum_{l=1}^{S} \frac{1}{\delta(\hat{\vec{g}}_l)}}.$$
 (17)

Basing on the calculated probabilities, we determine the number of points for the study of the possible local minima of the target function from problem (12) by formula (2.22). However, assuming that value  $m_l$  in this formula should be integer, since it determines the number of points in the vicinity of the studied point in order to find the minimum of the goal function, formula (17) will be rewritten as follows:

$$m_l = ToInt(P_l \cdot S), \ l = 1, ..., S, \ m_{l=1} = 0,$$
 (18)

where  $ToInt(\cdot)$  is the operator of the selection of the integer part of the number.

Then the procedure is repeated to determine the points where the lowest value of the goal function is achieved.

To avoid focusing on the local minima of the target function, we use the phase of bee-scouts.

Phase of bee-scouts. This is the phase where new solutions of the optimization problem are again randomly produced. To do this, formula (12) is used. As it is noted above, in the context of the bee colony's behavioral model, this means depletion of current nectar sources.

Each iteration of calculations involves obtaining a new number of points in addition to the current ones. At the end of each iteration we have 2S points being candidates for research. Therefore, at the end of the iteration, we perform a group selection of points with the smallest value of the goal function  $\delta(\hat{g}_l)$  so that their number is equal to the value of S. This procedure is called group selection.

## 4. Ontological approach

The difficulty in applying the above approach is the complexity of using the constructed interval model in the case of other environmental conditions. For example, in the work [3] the application of this method to build a model for the spread of pollutants from vehicles using modeling of the daily cycle of the dynamics of concentrations of harmful emissions of NO2 (nitrogen dioxide) at the intersection of Ruska - Zamkova - Shashkevycha, Ternopil. The results of experiments showed that the model of dynamics of nitrogen dioxide concentrations constructed in the paper is not universal, i.e., it cannot be used for modeling in other parts of the city, but in other weather conditions. This fact is caused by disregard for the action of various, including random factors that are specific to a particular point at which the simulation is carried out. This type of factor includes natural ventilation, the presence of vertical and horizontal air flows, etc.

Thus, to improve the predicted properties of the models, we propose modifying the above approach using a nonlinear element in the model which acts as a switch to select an adequate model, depending on the conditions of its application.

Fig. 1 shows the flow chart of interaction between external information systems, modeling environment, information repository and ontologically controlled subsystem.

The information repository stores a database of developed mathematical models which includes the following main components: formalized mathematical models (equations, functions, parameters, constants); graphical interpretation of modeling results (graphs, tables, various schemes); results of experimental research (description of the subject area, characteristics of the experiments, the results of the models).

An ontologically controlled SmartOntologyModeller software system includes the implementation of declarative and operational ontology of mathematical modeling based on interval analysis and contains a subsystem of interaction with external information systems and a subsystem of communication with information repository to reuse mathematical models and build new ones.

The model of interaction of software subsystems serves to obtain information about the principles of interaction arising in the process of construction and use of mathematical models. This information includes the data about the interaction protocols, the structure of the packets for data exchange and the rules for managing the relevant exchange.

An important element of this approach is the ontological model of the subject area which takes into account the different conditions of application of interval models. Consider this approach in more detail.

The ontologically controlled system interprets the knowledge gained in the process of using interval mathematical models. This knowledge can be divided into declarative and procedural parts.

The declarative part consists of the information necessary for the implementation of the solution of the model, the information obtained from the model and the corresponding equations of the model.

The procedural part consists of detailed features of the model, the corresponding used methods and algorithms for finding solutions, initialization of variables and their corresponding interpretation.

The ontological approach provides a model of representation of the subject area and serves as a kind of nonlinear element of the interval discrete model, built on the analysis of interval data to expand the scope of simpler models in a particular subject area.



Fig. 1. The flow chart of realization of interaction of ontologically controlled system with external information systems and the corresponding information repository

Fig. 2 shows the flow chart of selection of the nonlinear element of the mathematical model, taking into account its main features. Let us consider these features in more detail.

The process of selection of a nonlinear element must be in the domain of its main characteristics and the characteristics of the mathematical model. The main characteristics of the nonlinear element include:

 generating parameters, which can contain both input and output data. These parameters can be obtained both from the information repository and in the process of interaction with external information systems;

 hypotheses that describe the main options for using the selected nonlinear parameter, as well as other specific assumptions; - objects that allow highlighting the main properties of the studied characteristics and forming the main objectives of its impact;

actions that determine the possible set of operations for the use of the element;

 a platform that characterizes a set of software that can be used in the implementation of a mathematical model;

- subject area which characterizes the set of interconnected functions, management tools, considered within a certain context.

The main characteristics of the mathematical model based on the interval approach which would correlate with the characteristics of the selected nonlinear element include the following:

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Fig. 2. Flow chart of selection of a nonlinear element of an intrinsic mathematical model taking into account its main features and model features

– functions which along with the implementation of generally accepted mathematical calculations include the implementation of the basic processes of describing the mathematical model; it is also necessary to take into account the degree of understanding and comprehension of the functions that are taken into account at the stage of selecting possible variants of usage;

 usage that allows describing and evaluating main features related to a particular subject area and external information systems, including external modeling environments;

technical characteristics, which include quantitative (error, probability) and qualitative (accuracy, adequacy, complexity) evaluation of models in the process of their construction and practical use.

A very important stage of the separation of a nonlinear element of the interval discrete model is the process of its coordination as a separate object with its inherent characteristics and the mathematical model itself which has its own set of corresponding characteristics.

This process can be formalized on the basis of appropriate formal descriptions of the ontological model of the subject area and the ontological approach to modeling based on interval data. The rules and operators of consistency of the mentioned characteristics can bedescribed on the basis of operations of the classical algebra of tuples.

Another feature of the proposed approach is that the components of the model created in this way are completely reusable, i.e., equations, variables, assumptions from one model can be reused when creating another model, or the resulting database of mathematical models can be reused in the interpretation in other information systems.

The control subsystem of switching the nonlinear parameter in the process of selecting mathematical models initializes operators for the formation of model parameters with appropriate values, creates associations between variables, forms actual commands to build the model and provides the corresponding solution of the set of equations.

It is also possible to simulate the flow of certain operations which is quite convenient for making decisions. Thus, methods and means of constructing discrete models of objects can be combined with distributed parameters in the form of interval difference equations based on a combination of ontological approach and analysis of interval data.

#### 5. Experimental studies

The practical implementation of the ontology of mathematical modeling on the basis of interval data leads to the formation of common structural elements, based on the peculiarities of their use for specific subject areas. In the paper, the practical implementation of software is realized as one of the options for using the developed base of model experiments in various subject areas within the proposed ontological approach.

The ontological approach allowed building the models of representation of the subject area, and the proposed flow chart of selecting a nonlinear element of the interval discrete model built on the analysis of interval data allowed expanding the scope of simpler models in the selected subject area.

A number of experimental studies were carried out on the software implementation of the method of switching by a nonlinear element of a mathematical model depending on changes in the conditions of the course or factors of experiments.

The system is implemented using the Java programming language, Python connection tools and MySQl relational database.

The user chooses a certain model and a corresponding operational example. For the model chosen, a nonlinear element is selected which, based on consistency rules, allows building appropriate operators using appropriate libraries that interpret equations from formatted, indexed parts, initialize appropriate model parameters based on the corresponding instance of the operation and, finally, choose the model to provide the necessary solution or to refine and build a new model.

The obtained results can be interpreted in the appropriate graphical interface (graphs, tables, diagrams, set of resulting files), and other results are also displayed, which are then stored back in the operational instance of the mathematical model with appropriate clarifications and placed in the information repository with a separate identifier. This clarification will in the future allow choosing the right models depending on the specifics of the selected conditions or changes in significant conditions of use in the relevant subject area.

Let us consider an example of application of the ontological approach for construction of interval discrete dynamic model for modeling of a daily cycle of dynamics of concentrations of harmful emissions of nitrogen dioxide concentrations at one of intersections with the heavy traffic of motor transport in Ternopil.

Several interval mathematical models have been found in the information repository for this research object. Fig. 3 shows a formal representation of the selected model in the SmartOntologyModeller environment.

The model shown in Fig. 3 represents the relationship between the concentration of nitrogen dioxide and the intensity of traffic for different parts of the city.

As a result of the analysis of the data sample for this example, it was possible to determine the reason for the impossibility of building a single model to study the dynamics of nitrogen dioxide concentrations at intersections due to traffic pollution, namely due to ambiguity between nitrogen dioxide concentration and traffic intensity for different points of the city. According to the analysis, to eliminate this problem it is necessary to control the selected structural element of the model.

	Conics	Mathematical description of the model
8	C) Elses	$[\bar{v}_k] = 0.0365 + 0.3542 \cdot [\bar{v}_{k-1}] + 0.118 \cdot [\bar{v}_{k-1}] \cdot [\bar{v}_{k-3}] +$
		• +0,5059 $\cdot [\bar{v}_{k-1}] \cdot u_{1,k} / u_{1,k-1} - 0.0154 \cdot [\bar{v}_{k-2}] \cdot u_{1,k-1} / u_{1,k+1}$
	Transform	Choose model
	Transations/	intervals of predicted values of hitrogen dicxide concentrations
	Andrew Market	Intervals of measured values of concentrations of netrogen dioxide

## Fig. 3. Choice of interval mathematical model for modeling the daily cycle of the dynamics of concentrations of harmful emissions of nitrogen dioxide in the environment SmartOntologyModeller

Fig. 4 shows the procedure for selecting a nonlinear element of an interval discrete dynamic model and the corresponding switching to another instance of the model. Thus, the results obtained within the ontological approach and analysis of interval data enable us to represent the subject area as a complex nonlinear element of the interval discrete model built on the analysis of interval data to expand the scope of simpler models in the selected subject area.

## 6. Conclusions

Despite the current progress in most areas of use and control of the process of mathematical modeling, knowledge control in this area is a very promising area of research and practice-oriented research.

In the framework of this work the approach to construction of mathematical and computer models with application and combination of methods of the analysis of interval data and methods of ontological analysis for the purpose of increase of prognostic characteristics of models is offered.

This approach greatly simplifies the process of creating a mathematical model by manipulating the nonlinear element of the interval discrete model for reusing it, improves the predicted characteristics of the models and testing in appropriate external softwareinterpreted environments.

In further research, it is planned to scale the developed software environment for modeling objects with distributed parameters in the form of interval difference equations based on a combination of the ontological approach and the analysis of interval data.



Fig. 4. Results of selection of a nonlinear element of a discrete dynamic model and corresponding switching to another instance of a mathematical model in the SmartOntologyModeller environment

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## УПРАВЛІННЯ ПРОЦЕСОМ МАТЕМАТИЧНОГО МОДЕЛЮВАННЯ ДИНАМІКИ КОНЦЕНТРАЦІЙ ШКІДЛИВИХ РЕЧОВИН НА ОСНОВІ ОНТОЛОГІЧНОГО ПІДХОДУ

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Розглянуто проблему побудови математичної моделі динаміки концентрацій діоксиду азоту на різних ділянках міста, особливості побудови таких моделей на основі періодичного вимірювання концентрацій шкідливих речовин та ідентифікації на підставі отриманих вимірювань. У статті також запропоновано онтологічний підхід як інструмент управління, що дає змогу значно спростити систематичні стандартизовані методи зберігання моделей, процес їх побудови та відповідного використання у практичних ситуаціях.

Використання онтологічної моделі дає змогу формалізувати процес отримання, зберігання та використання відповідних знань та підходить для інтелектуалізованіших систем, забезпечує визначення завідомо хибних розв'язків на основі моделі, прогнозного контролю моделі, оптимізації процесу прийняття рішень на основі знань та моделювання відповідної технологічної схеми.

Описано також особливості побудови відповідної онтологічної моделі, схему вибору нелінійної моделі з "перемиканнями" на різні умови. В роботі висвітлено також відповідні експериментальні дослідження, які дають змогу підтвердити ефективність запропонованого підходу.





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