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MATHEMATICAL MODELING OF STATIONARY THERMOELASTIC STATE IN A HALF PLANE CONTAINING A PERIODIC SYSTEM OF CRACKS DUE TO PERIODIC LOCAL HEATING BY A HEAT FLUX

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Abstract. *Purpose*. To determine the two-dimensional thermoelastic state in a semi-infinite solid (half-plane), weakened by a system of periodic internal cracks under conditions of local heating on the edge of the half plane. Heat flux due to frictional heating on the local area of the body, causes changes in temperature and stresses in the body, which significantly affects its strength, as it can lead to crack growth and local destruction. Therefore, the study of the problem of frictional heat is of a practical interest. This paper proposes to investigate the stress-deformed state in the vicinity of the crack tip, depending on the period of cracks placement.

Methodology. The methods for studying two-dimensional thermoelastic state of a body with crack as stress concentrators are based on the method of complex variable function. Reducing the problem of stationary heat conduction and thermoelasticity to singular integral equations (SIE) of the first kind, the numerical solution by the method of mechanical quadrature was obtained.

Findings. In this paper, we present graphical dependencies of stress intensity factors (SIF) at the crack tip on the angle of orientation and on the relative position of cracks. The obtained results will be used later to determine the critical value of the intensity of the local heat flux from equations of limit equilibrium at which crack growth and the local destruction of the body occur.

Originality. The originality of our solution lies in the fact that the new two-dimensional problems of heat conduction and thermoelasticity for a half plane containing a periodic cracks due to local heating by a heat flux are obtained.

Practical value. The practical value is the ability to extend our knowledge of the real situation in the thermoelastic elements of engineering structures with cracks that operate under conditions of heat stress (frictional heat) in various industries, particularly in mechanical engineering. The results of specific values of SIF at the crack tip in graphs may be useful in the development of sustainable modes of structural elements in terms of preventing the growth of cracks.

Keywords: crack, heat flux, heat condition, thermoelasticity, stress intensity factor, singular integral equation.

Introduction

Elements of many modern structures are often designed for their operation under conditions of thermal heating; such conditions contribute to the emergence of their thermal stresses. This is typical for tools and structures in the heat industry. The level of concentration and intensity of these stresses in some areas, for example, in the neighborhood of non-homogeneities (cracks, inclusions) of technological nature largely determine their operability. Therewith, the fracture of materials is caused by the presence of sharp concentrators of stresses such as cracks. Therefore, the study of thermoelastic state near a crack is necessary for calculations of strength from the point of view of fracture mechanics, which is especially important for structures of high strength and little plastic materials, which are under the action of different kinds of heat loads.

The multiple cracks problem in an elastic half plane is formulated into a singular integral equation using the modified complex potential with free traction boundary condition. A system of singular integral equations is obtained with the distribution dislocation function as unknown, and the traction applied on the crack faces and the right hand terms in the paper [1]. In the paper [2], the problem of the stress concentration in the vicinity of the crack tips for a crack of finite length located perpendicular to the interface of two elastic bodies – a half plane and a strip is considered. Using the method of generalized integral transform, the problem reduces to solution of a singular integral equation with a Cauchy kernel. Values of the stress intensity factors of the normal stresses in the vicinity of crack tips are obtained for different combinations of the geometrical and physical parameters of the problem.

The two-dimensional problems of thermoelasticity for semi-infinite bodies with cracks have already been investigated in the literature. Thus, in particular, the thermoelastic state of a half plane containing an internal rectilinear crack at different temperature and force conditions imposed on the crack lips and on the edge of the half plane were analyzed in [3], [4], [5]. The method of SIE was used for the analysis of the plane thermoelastic state in a half space containing an internal arbitrary oriented rectilinear crack [6], [7], internal curvilinear crack [8], edge crack [9], an inclusion and a crack [10] due to local heating over a part of its free surface by a heat flux.

In this paper, a thermal problem for thermoelastic half plane, weakened by a system of periodic internal cracks under conditions of local heating on the edge of the half plane is considered. This model is the development of the previous models to determine the two-dimensional thermoelastic state in a semi-infinite solids, weakened by a system of internal cracks.

Problem statement

Consider an elastic half plane containing a periodic system of cracks when in the main band of periods of width d (along the axis Ox) there are N of internal rectilinear cracks $L_k \left(k=\overline{1,N}\right)$. We assume that all contours $L_k \left(k=\overline{1,N}\right)$ do not have common points. Each contour is connected with a local coordinate system $x_k O_k y_k$ whose axis $O_k x_k$ forms an angle α_k with the axis Ox, and the points O_k are determined in the coordinate system xOy by complex coordinates z_k^0. The problem is studied under the conditions of a stationary temperature field.

Consider the problem of heat conduction with the following conditions of thermal contact .We will assume that the cracks lips are thermally insulated:

$$\frac{\partial T^{\pm}}{\partial n}=0, \qquad t_k \in L_k, \quad k=\overline{1,N}. \tag{1}$$

Here n is the outer normal to the left face of the cut $L_k \left(k=\overline{1,N}\right)$; $T(x,y)$ stands for the temperature; t_k are the complex coordinates of points on the contours L_k in local coordinate systems; the plus and minus indices indicate the boundary values of the corresponding values on the left and on the right of the approach to the contour L_k. In the problem of thermoelasticity, we suppose that a domain of bounded width $2a$ located on the edge on the half plane is heated by a periodic heat flux with intensity q. The other parts of the edge of the half plane are thermally insulated. Assume that the cracks lips are not in contact and are free of the loads in the process of deformation:

$$[N(t_k)+iS(t_k)]^{\pm}=0, \qquad t_k \in L_k, \quad k=\overline{1,N}. \tag{2}$$

In the relations (2), $N(t_k)$ and $S(t_k)$ are the normal and the tangential components of the load.

Problem solution. System of integral equations of the problem of heat conduction

We represent the total temperature in the half plane with a periodic system of cracks in the form:

$$T(x,y)=T_0(x,y)+T^*(x,y),$$

where $T_0(x,y)$ is the temperature in the homogeneous half plane without cracks caused by the periodic heat flux of intensity q and $T^*(x,y)$ is the perturbed temperature generated by cracks. The temperature field $T_0(x,y)$ is described in [7].

We represent the temperature $T^*(x,y)$ in the form of $T_*(x,y)=\operatorname{Re} f(z)$, and we use the complex temperature potential $F(z)=f'(z)$ [11]:

$$F(z)=F_0(z)+\frac{1}{id}\sum_{k=1}^{N}\int_{L_k}\left[H_k(t_k)ctg\left[\frac{\pi}{d}(\zeta_k-z)\right]dt_k-\overline{H_k(t_k)}ctg\left[\frac{\pi}{d}(\bar{\zeta}_k-z)\right]dt_k\right], \tag{3}$$

$$H_k(t_k)=\gamma_k'(t_k), \qquad \zeta_k=t_k+z_k^0.$$

The function $F_0(z)$ determines the temperature field $T_0(x,y)$ in a continuous homogeneous half plane, and the function $T_0(x,y)$ is periodic with respect to the coordinate x with period d.

By satisfying the boundary conditions (1) with the help of the complex temperature potential (3), we obtain in the heat conduction problem the system of N singular integral equations of the first kind for the N unknown functions $\gamma_k'(t_k)$ $\left(k=\overline{1,N}\right)$ on the contours of cracks:

$$\frac{1}{\pi}\sum_{k=1}^{N}\int_{L_k}\operatorname{Im}\left[K_{nk}(t_k,\tau_n)H_k(t_k)dt_k-L_{nk}(t_k,\tau_n)\overline{H_k(t_k)dt_k}\right]=$$

$$=\operatorname{Im}\left\{\left[(F_0(\eta_n)\right]e^{i(\alpha_n)}\right\}^*; \qquad \tau_n\in L_n, \quad n=\overline{1,N}. \tag{4}$$

Here:

$$K_{nk}(t_k,\tau_n)=\frac{\pi}{id}e^{i\alpha_n)}ctg\left[\frac{\pi}{d}(\zeta_k-\eta_n)\right];$$

$$L_{nk}(t_k,\tau_n)=\frac{\pi}{id}\frac{e^{i\alpha_n)}}{i(\bar{\zeta}_k-\eta_n)}ctg\left[\frac{\pi}{d}\left(\bar{\zeta}_k-\eta_n\right)\right]; \qquad \eta_n=\tau_n+z_n^0;$$

$$F_0(z)=\frac{\partial T_0(x,y)}{\partial x}-\frac{i\partial T_0(x,y)}{\partial y}.$$

The solution of system integral equations (4) must satisfy the condition:

$$\int_{L_n}\gamma_n'(t_n)dt_n=0, \quad n=\overline{1,N}, \tag{5}$$

which provide continuity of temperature by passing the contours of crack. Under condition (5), the system of integral equations (4) for an arbitrary its right-hand side has a unique solution.

System of integral equations of the problem of thermoelasticity

Since the stationary temperature field $T_0(x,y)$, in the absence of internal heat sources, does not generate a stresses in a homogeneous simply connected half plane [13], we find the stresses generated by the perturbed temperature $T^*(x,y)$.

Complex stress potentials can be represented in the form [11]:

$$\Phi(z)=\Phi_1(z)+\Phi_2(z), \ \Psi(z)=\Psi_1(z)+\Psi_2(z), \tag{6}$$

where

$$\Phi_1(z)=\frac{1}{2d}\sum_{k=1}^{N}\int_{L_k}Q_k(t_k)\operatorname{ctg}\left[\frac{\pi}{d}(\zeta_k-z)\right]e^{i\alpha_k}dt_k, \quad \zeta_k=t_ke^{i\alpha k}+z_k^0; \tag{7}$$

$$\Psi_1(z)=\frac{1}{2d}\sum_{k=1}^{N}\int_{L_k}\left\{\overline{Q_k(t_k)}\operatorname{ctg}\left[\frac{\pi}{d}(\zeta_k-z)\right]e^{-i\alpha_k}\overline{dt_k}-\right.$$

$$\left.-\left\{\operatorname{ctg}\left[\frac{\pi}{d}(\zeta_k-z)\right]+\frac{\pi}{d}\left(\overline{\zeta_k}-\zeta_k+z\right)\operatorname{cosec}^2\left[\frac{\pi}{d}(\zeta_k-z)\right]\right\}Q_k(t_k)e^{i\alpha_k}dt_k\right\};$$

$$\Phi_2(z)=\frac{1}{2d}\sum_{k=1}^{N}\int_{L_k}\left\{Q_k(t_k)\operatorname{ctg}\left[\frac{\pi}{d}\left(z-\overline{\zeta}_k\right)\right]e^{i\alpha_k}dt_k+\right.$$

$$\left.+\frac{\pi}{d}\left(\zeta_k-\overline{\zeta}_k\right)\operatorname{cosec}^2\frac{\pi}{d}\left(\overline{\zeta}_k-z\right)\overline{Q_k(t_k)}e^{-i\alpha_k}\overline{dt_k}\right\};$$

$$\Psi_2(z)=\frac{1}{2d}\sum_{k=1}^{N}\int_{L_k}\left\{Q_k(t_k)\left[\operatorname{ctg}\left[\frac{\pi}{d}\left(\overline{\zeta_k}-z\right)\right]+\frac{\pi z}{d}\operatorname{cosec}^2\left[\frac{\pi}{d}\left(\overline{\zeta}_k-z\right)\right]\right]e^{i\alpha_k}dt_k+\right.$$

$$+\left\{\operatorname{ctg}\left[\frac{\pi}{d}\left(z-\overline{\zeta_k}\right)\right]+\left[\frac{2\pi z}{d}\operatorname{ctg}\left[\frac{\pi}{d}(\zeta_k-z)\right]+1\right]\times$$

$$\left.\times\frac{\pi}{d}\left(\overline{\zeta_k}-\zeta_k\right)\operatorname{cosec}^2\left[\frac{\pi}{d}\left(\overline{\zeta_k}-z\right)\right]\right\}\overline{Q_k(t_k)}e^{-i\alpha_k}\overline{dt_k}\right\};$$

$$Q_k(t_k)=g_k'(t_k)+\frac{i\beta_*}{1+\chi_*}\left[f^+(t_k)-f^-(t_k)\right],\quad t_k\in L_k,\ k=\overline{1,N}.$$

The functions $f^{\pm}(t_k)$ are defined as the limiting values of potential $f(z)$. The unknown function $g_k'(t_k)$, which is the derivative of the jump of displacements in passing through the cracks contour, is sought in the class of Hölder function with integrable singularities at the crack tips.

Here are the following designations: $\chi=(3-\mu)/(1+\mu)$, $\beta^t=\alpha^t E(1+\mu)$ are for a plane stressed state; α^t, G, E are the coefficients of linear thermal expansion, shear modulus, Young's modulus, μ is Poisson's ratio material of the matrix.

By satisfying the boundary conditions (2), with the help of the complex potentials (6), we obtain in the problem of thermoelasticity the system of N singular integral equations of the first kind for the N unknown functions $Q_k(t_k)$ $\left(k=\overline{1,N}\right)$ on the contours of cracks:

$$\frac{1}{2\pi}\sum_{k=1}^{N}\int_{L_n}\left[R_{nk}(t_k,\tau_n)Q_k(t_k)dt_k+S_{nk}(t_k,\tau_n)\overline{Q_k(t_k)}\,\overline{dt_k}\right]=0,\quad \tau_n\in L_n,\ n=\overline{1,N}. \qquad (8)$$

Here:

$$R_{nk}(t_k,\tau_n)=R^1_{nk}(t_k,\tau_n)-e^{i\alpha_k}\times\left\{B_n ctg\frac{\pi}{d}\left(\eta_n-\overline{\zeta}_k\right)+\right.$$

$$+C_n e^{-2i\alpha_n}ctg\left[\frac{\pi}{d}\left(\eta_n-\overline{\zeta}_k\right)\right]+\frac{\pi}{d}\left(\overline{\eta_n}-\eta_n\right)\operatorname{cosec}^2\left(\overline{\eta_n}-\zeta_k\right)\cdot C_n\frac{d\overline{\tau_n}}{d\tau_n}\times$$

$$\left.\times\left[1-e^{-2i\alpha_n}+2\frac{\pi}{d}e^{-2i\alpha_n}\left(\overline{\eta_n}-\eta_n\right)\operatorname{ctg}\left(\overline{\eta_n}-\zeta_k\right)\right]\right\};$$

$$S_{nk}(t_k,\tau_n)=S^1_{nk}(t_k,\tau_n)+\frac{\pi}{2d}e^{-i\alpha_k}\left\{-B_n\frac{\pi}{d}\left(\overline{\eta_n}-\eta_n\right)\operatorname{cosec}^2\left(\eta_n-\overline{\zeta}_k\right)+\right.$$

$$\left.+\left(1-e^{-2i\alpha_k}\right)C_n\cdot\operatorname{ctg}\left[\frac{\pi}{d}\left(\overline{\eta_n}-\zeta_k\right)\right]+\frac{\pi}{d}C_n e^{-2i\alpha k}\left(\overline{\eta_n}-\eta_n\right)\operatorname{cosec}^2\left[\frac{\pi}{d}\left(\overline{\eta_n}-\zeta_k\right)\right]\right\};$$

$$R^1_{nk}(t_k,\tau_n)=\frac{\pi}{2d}\left\{e^{i\alpha_k}B_n\operatorname{ctg}\left[\frac{\pi}{d}(\zeta_k-\eta_n)\right]+e^{-i\alpha_k}C_n\frac{d\overline{\tau_n}}{d\tau_n}\operatorname{ctg}\left[\frac{\pi}{d}\left(\overline{\zeta_k}-\overline{\eta_n}\right)\right]\right\};$$

$$S^1_{nk}\left(t_k,\tau_n\right)=\frac{\pi}{2d}\cdot C_n\left(e^{-i\alpha_k}-e^{-3i\alpha_k}\right)\cdot\left\{\operatorname{ctg}\left[\frac{\pi}{d}\left(\overline{\zeta_k}-\overline{\eta_n}\right)\right]-\frac{\pi}{d}\left(\overline{\zeta_k}-\overline{\eta_n}\right)\times\right.$$

$$\left.\times\frac{\overline{d\tau_n}}{d\tau_n}\operatorname{cosec}^2\left[\frac{\pi}{d}\left(\overline{\zeta_k}-\overline{\eta_n}\right)\right]\right\};\qquad \eta_n=\tau_n e^{i\alpha_n}+z_n^0 .$$

In the class of functions $g'_k\left(t_k\right)\in H^*$ $\left(k=\overline{1,N}\right)$ the system of integral equations (8) has a unique solution for an arbitrary its right-hand side under the conditions:

$$\oint_{L_n} g'_n\left(t_n\right)dt_n=0\,,\quad n=\overline{1,N}\,. \tag{9}$$

Conditions (9) ensure the uniqueness of displacements by passing the contours of cracks. By using relation (7), we represent condition (9) for thermally insulated cracks in the form:

$$\oint_{L_n} Q_n(t_n)\,dt_n=-\frac{2i\beta^t}{1+\chi}\oint_{L_n} t_n\gamma'(t_n)\,dt_n\,,\quad n=\overline{1,N}\,. \tag{10}$$

The algorithm used for the solution of the analyzed problem can be described as follows: the system of integral equations (4), (5) of the problem of heat conduction is used to find the functions $\gamma'_k\left(t_k\right)$ $\left(k=\overline{1,N}\right)$. These functions are then substituted in system of equations (8), (10) of the problem of thermoelasticity to find the unknown functions $Q_k\left(t_k\right)$ $\left(k=\overline{1,N}\right)$. Then the stress intensity factors (SIF) K_I and K_{II} are found according to the formula [11]:

$$K_I^{\pm}-iK_{II}^{\pm}=\mp\lim_{t_k\to l_k^{\pm}}\left[\sqrt{2\pi\left|t_k-l_k^{\pm}\right|}\,Q_k\left(t_k\right)\right],\quad k=\overline{1,N}\,, \tag{11}$$

where the lower signs correspond to the beginning of the crack $\left(l_k^-\right)$, and the upper ones correspond to the end of the crack $\left(l_k^+\right)$.

System of single periodic cracks

Consider a half plane with a periodic system of internal cracks when in the main band of the period there is one rectangular crack at an angle α to the axis Ox. The centers of the cracks are on one straight line $y=-h$, where h is the distance from the centers of the cracks to the edge of the half plane. The distance between the centers of neighborhood cracks is the same and equal to d_0. Let the edge of the half plane be locally heated by a uniform heat flux $q=\mathrm{const}$ on the section in width $2a$, and this section changes periodically along the coordinate Ox with the period d_0. The edge of the half plane outside the area of heating and the lips of the crack are insulated and free from external forces. Then the basic temperature field $T_0\left(x,y\right)$ in the half plane without cracks changes periodically along the coordinate x with the period d_0. The center of the coordinate system $O'x'y'$, where $x'=x+d$, $y'=y$ is chosen in the middle of a section of heating distant from the axis Oy by distance d (Fig. 1).

The stationary temperature field of the thermoelastic half plane without cracks caused by the heat flux q takes the form [7]:

$$T_0\left(x,y\right)=-\frac{1}{\pi\lambda}\int_{-a}^{a} q\ln\sqrt{\left(x'-\xi\right)^2+y'^2}\,d\xi=$$

$$=\frac{q}{\pi\lambda}\left\{\left(x+d-a\right)\ln\sqrt{\left(x+d-a\right)^2+y^2}-\left(x+d+a\right)\ln\sqrt{\left(x+d+a\right)^2+y^2}+\right. \tag{12}$$

$$\left.+y\left[arctg\left(\frac{x+d-a}{y}\right)-arctg\left(\frac{x+d+a}{y}\right)\right]+2a\right\}.$$

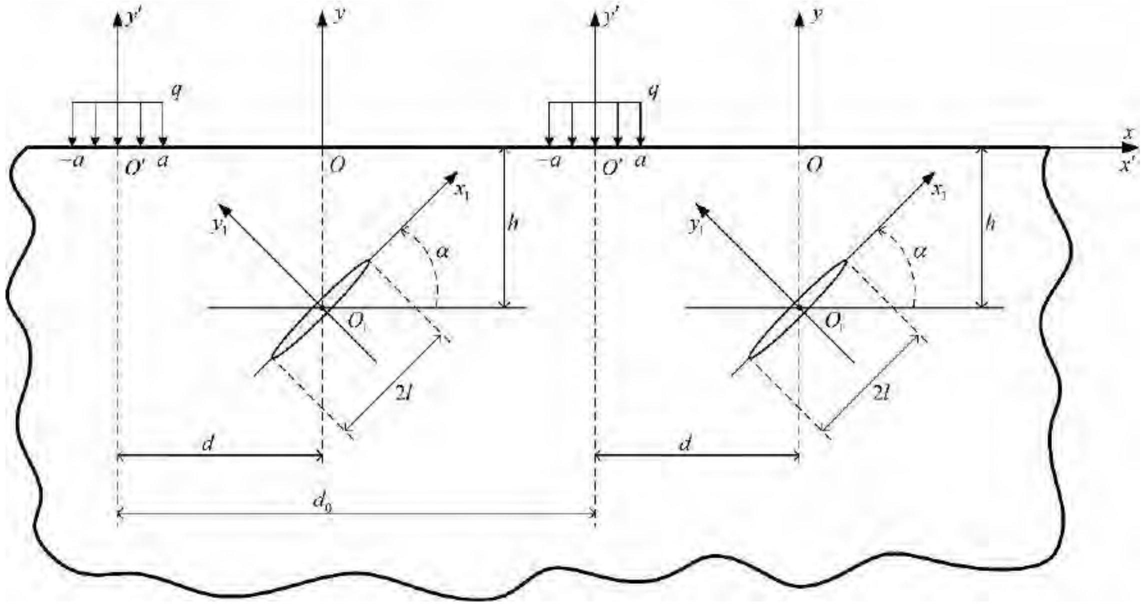

Fig. 1. System of single periodic cracks and heat fluxes

To found of the disturbed temperature $T^*(x,y)$ generated by cracks, integral equation in the heat conduction problem takes the form:

$$\frac{1}{\pi}\int_{-l}^{l}\left[\frac{1}{t_1-x_1}+L(t_1,x_1)\right]g'(t_1)\,dt_1=F(x_1),\quad |x_1|<l, \tag{13}$$

with the kernel [11]:

$$L(t_1,x_1)=\frac{\pi}{d_0}\mathrm{Re}\left\{e^{i\alpha}ctg\left[\frac{\pi}{d_0}(t_1-x_1)e^{i\alpha}\right]\right\}+\frac{\pi}{d_0}\mathrm{Re}\left\{e^{i\alpha}ctg\left[\Omega(x_1,t_1)\right]\right\};$$

$$\Omega(x_1,t_1)=\frac{\pi}{d_0}\left(x_1e^{i\alpha}-t_1e^{-i\alpha}-2ih\right);$$

$$F(x_1)=-\left.\frac{\partial T_0(x,y)}{\partial y_1}\right|_{y_1=0}=\sin\alpha\left.\frac{\partial T_0(x,y)}{\partial x}\right|_{y_1=0}-\cos\alpha\left.\frac{\partial T_0(x,y)}{\partial y}\right|_{y_1=0}.$$

Additionally, the solution of integral equation (13) has to satisfy the condition of continuity of temperature $T^*(x,y)$ around the crack contour:

$$\int_{-l}^{l}g'(t_1)\,dt_1=0. \tag{14}$$

The integral equation of the problem of thermoelasticity can be written in the form:

$$\int_{-l}^{l}\left[R(t_1,x_1)Q(t_1)+S(t_1,x_1)\overline{Q(t_1)}\right]dt_1=0,\quad |x_1|<l, \tag{15}$$

where the kernels are given as [11]:

$$\begin{aligned}R(t_1,x_1)={}&0.5\frac{\pi}{d_0}e^{i\alpha}ctg\left[\frac{\pi}{d_0}(t_1-x_1)e^{i\alpha}\right]+e^{-i\alpha}ctg\left[\frac{\pi}{d_0}(t_1-x_1)e^{-i\alpha}\right]+0.5\frac{\pi}{d_0}e^{i\alpha}\times\\&\times\left\{ctg\Omega(x_1,t_1)+e^{-2i\alpha}ctg\overline{\Omega(x_1,t_1)}-2\frac{\pi i}{d_0}(t_1\sin\alpha-h)\cos ec^2\overline{\Omega(x_1,t_1)}\times\right.\\&\left.\times\left[1-e^{-2i\alpha}-4\frac{\pi}{d_0}e^{2i\alpha}(x_1\sin\alpha-h)ctg\overline{\Omega(x_1,t_1)}\right]\right\};\end{aligned} \tag{16}$$

$$S(t_1,x_1)=0.5\frac{\pi}{d_0}\left(e^{-i\alpha}-e^{-3i\alpha}\right)\left[\frac{2\pi i}{d_0}(t_1\sin\alpha-h)\cos ec^2\Omega(x_1,t_1)+\right.$$

$$\left.+\left(1-e^{-2i\alpha}\right)ctg\overline{\Omega(x_1,t_1)}-2\frac{\pi i}{d_0}e^{-2i\alpha}\left(x_1\sin\alpha-h\right)\cos ec^2\overline{\Omega(x_1,t_1)}\right];$$

$$Q(t_1)=g'(t_1)+2i\beta^t\gamma(t_1)/(1+\chi),\quad \beta^t=\alpha^t E.$$

By using relation (16), we represent the condition (10) for insulated thermally crack in the form:

$$\int_{-l}^{l}Q(t_1)dt_1=-\frac{2i\beta^t}{1+\chi}\int_{-l}^{l}t_1\gamma'(t_1)\,dt_1,\qquad(17)$$

which ensures the uniqueness of displacements by passing the contour of crack.

Numerical analysis

The solution algorithm of considered problem can be described as follows:

1. From the integral equation (13) and the condition (14) the derivative of temperature jump $\gamma'(t_1)$ is calculated;

2. The next, having the function $\gamma'(t_1)$ the integral equations (15), (17) will be solved with respect of function $Q_1(t_1)$;

3. Then the stress intensity factors (SIFs) K_I and K_{II}, which are the real quantities that characterize the stress-deformed state in the vicinity of the crack tips, are found according to the formula:

$$K_I^{\pm}-iK_{II}^{\pm}=\pm\lim_{t_1\to\pm l}\sqrt{2\pi\left|t_1\mp l\right|}\,Q(t_1).\qquad(18)$$

The numerical solution of the system equations (13), (14), and (15), (17) are found by the method of mechanical quadratures [13].

The calculations of the dimensionless stress intensity factors (SIFs)

$$k_1^{\pm}=K_{\mathrm{I}}^{\pm}\lambda(1+\chi)/q\beta_t l\sqrt{l}\qquad\text{and}\qquad k_2^{\pm}=K_{\mathrm{II}}^{\pm}\lambda(1+\chi)/q\beta_t l\sqrt{l}$$

are derived for the cracks system normally to the boundary half plane ($\alpha=90°$). In this case, it follows that $k_1^{\pm}=0$, and the dependencies of $k_2^{\pm}$ on the dimensionless period $d_0^*=d_0/h$ for dimensionless parameters $a^*=a/h=0.25;\ d^*=d/h=1;\ l^*=l/h=0.5;\,0.9$ are shown in Fig. 2.

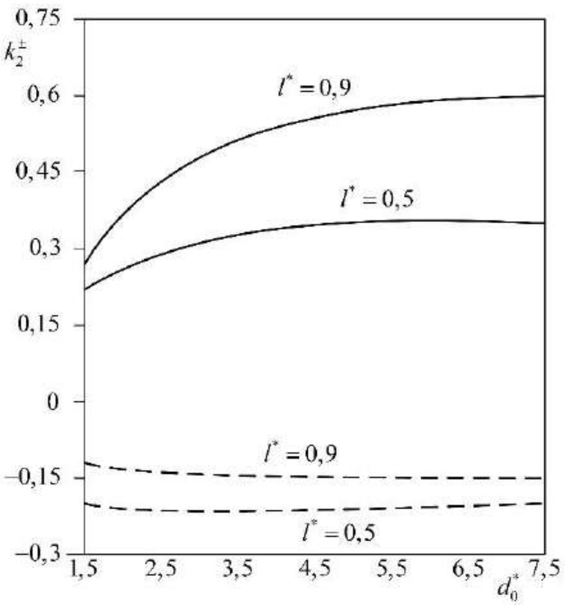

Fig. 2. The dependencies of the dimensionless SIFs $k_2^{\pm}$ on the dimensionless period d_0^* for $\alpha=90°$

The dashed lines correspond to the lower $l^* = 0.5$ tip of the crack (k_1^-, k_2^-), whereas the solid lines correspond to its upper tip (k_1^+, k_2^+). The increasing of period d_0^* leads to an increasing of SIFs, particularly it can be observed for long cuts ($l^* = 0.9$). Moreover, close distributed cracks reciprocal unload one of other and this configuration is safer than in the case of single crack [7]. For mechanical loading this result was presented in [11].

According to the results presented in [7], for a single crack and $l^* = 0.5$ there are $k_2^+ = 0.635$, $k_2^- = -0.195$ as well as for $l^* = 0.9$ there are $k_2^+ = 0.635$, $k_2^- = -0.147$. These stress intensity factors are obtained with the accuracy of 1 % from Fig. 2 for the values of periods: $d_0^* = 6.3$ for k_2^+ and $d_0^* = 1.7$ for k_2^- under the assumption that $l^* = 0.5$; for $l^* = 0.9$ we have $d_0^* = 10$ for k_2^+ and $d_0^* = 3$ for k_2^-. So, the influence of period d_0 on the SIFs is observed more strongly in the case of long cracks at the tip near the boundary half plane.

In the considered problem, the lips of the crack are not in contact. Then, according to the σ_θ-criterion (based on the hypothesis of the initial growth of the crack) from equations of the boundary equilibrium [14] it is possible to find the critical values of the heat flux q_{cr} at which the growth of the crack and the local destruction of the body begin, according to the formula:

$$q_{cr} = K_q \frac{K_{1C}}{\cos^3 \frac{\theta_*}{2} \left(k_1^{\pm} - 3k_2^{\pm} tg \frac{\theta_*}{2} \right)}, \qquad K_q = \frac{(1+\chi)\lambda}{\alpha_t E l \sqrt{\pi l}},$$

where K_{1C} is a constant of the material that characterizes the resistance of the material to the destruction and is determined experimentally; θ_* is an angle of initial growth of the crack:

$$\theta_* = 2 \operatorname{arctg} \frac{k_1^{\pm} - \sqrt{\left(k_1^{\pm}\right)^2 + 8\left(k_2^{\pm}\right)^2}}{4k_2^{\pm}}.$$

In the partial case for $l^* = 0.9$ we obtain: if $d^* = 2.5$, then $q_{cr} = 1.6 K_q K_{1c}$, if $d^* = 4.5$ then $q_{cr} = 0.9 K_q K_{1c}$, if $d^* = 6.5$ then $q_{cr} = 0.8 K_q K_{1c}$; for $l^* = 0.5$ we obtain: if $d^* = 2.5$, then $q_{cr} = 2.2 K_q K_{1c}$, if $d^* = 4.5$ then $q_{cr} = 1.6 K_q K_{1c}$, if $d^* = 4.5$ then $q_{cr} = 1.4 K_q K_{1c}$. These results are obtained for upper tip of the crack.

Conclusions

1. The two-dimensional mathematical models of the problems of stationary heat conductivity and thermoelasticity for elastic half plane with a periodic system of internal cracks under conditions of local heating on the edge of the half plane in the form of system of singular integral equations (SIEs) of the first kind on the contour of cracks are constructed. This approach allows to obtain a numerical solution of the system SIEs by applying the method of mechanical quadratures.

2. A numerical solution of the system SIEs in the partial case of a half plane with a system of single periodic thermally insulated cracks under conditions of local heating on the boundary of the half plane by given periodic heat flux of constant intensity are obtained. Based on this solution, the stress intensity factors (SIFs) at the crack tips are calculated, which in the future will be used to determine the critical values of the heat flux at which the crack begins to grow.

3. Based on the analysis of the obtained critical values q_{cr} of the heat flux, it follows: if the period of width d_0 increases, then critical value of the heat flux q_{cr} decreases (at which the growth of the crack in

the left tip begins) and approaching the value as in the case of a single crack. A similar situation is observed for the lower tip of the crack.

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