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Solving of differential equations systems in the presence of fractional derivatives using the orthogonal polynomials

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The mathematical model of the gas motion in the pipelines for the case where unstable process is described by the fractional time derivative is constructed in the paper. The boundary value problem is formulated. The solution of the problem is founded by the spectral method on Chebyshev-Laguerre polynomials bases with respect to the time variable and Legendre polynomials with respect to the coordinate variable. The finding of the solution eventually is reduced to the system of algebraic equations. The numerical experiment is conducted.

Keywords: *mathematical model, gas motion in pipelines, spectral methods, orthogonal polynomials.*

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1. Introduction

The evolution of theory and methods of the mathematical and computer modelling of the processes and systems in various fields of human activity always based on the use of new ideas approaches and methods from the area of analysis, applied and computational mathematics. Nowadays the one of actual problems of modelling is the problem of the mathematical model accordance to the investigated object. Dynamic systems as an modelling object are traditionally studied by the classical mathematical analysis use, in particular, the apparatus of integro-differential equations on the ordinary and fractional derivatives. In classical analysis these derivatives have integer order. However, it has been established a long time ago [1–5] that a number of objects and processes behavior does not accord entirely to the used mathematical models. It cause the necessity of the refined models development and use. It in turn causes more and more attention to studying and using of the fractional derivatives mathematical analysis [6–11].

Differential equations in partial fractional derivatives being the generalization of the partial derivatives of integer order besides the theoretical curiosity to them also have a great practical meaning [12–16]. A lot of physical processes are described by the dynamic systems in which taking into account the process history is very important. The problem of right choice of mathematical models for describing of investigated objects is very actual because the obtained results authenticity will depend on this. The fractional derivatives use is one of the ways of the process memory effects counting. Their appliance area is much wider then the appliance area of integer order derivatives whereas the latter are their partial case. In mathematical terms the investigation of natural processes mathematical models using fractional derivatives is reduced to the solving of convolution type integro-differential equations. It's known that the such equations are easily solved using the integral Laplace transform, which converts the integral convolution to multiplication of the kernel images and desired solution of integral equation [1, 2, 15, 16]. The orthogonal separation use gives a significant effect for such problems.

With the development of the science and technology the requirements of the modelling of various kinds of objects and processes, in particular, the hydrocarbon transport processes, increases. The need to construct new models and optimize existing ones are due to the fact that the objects are constantly complicate. It leads to increasing in both technological and financial costs, in particular, for increasing of the prices on the energy sources. So there is the need of decreasing of the energy costs for performing of existent tasks. Along with the mathematical models adequacy the time of their implementation is very important parameter because the relevant processes management passes on these results basis. Despite the fact that there is a large number of both analytical and numerical methods for mathematical physics problems solving at present, not all of them meet the requirements of the problems.

The spectral methods are used for solving of a wide class of mathematics and mechanics problems, in particular for solving of mathematical physics problems. Their essence is that the functions included into the model are presented in the form of orthogonal series in accordance to the selected basis. Finding the solution is reduced to calculating the coefficients of orthogonal series of the desired solution in this case. Nowadays there are few papers the solutions of mathematical physics problems are found in orthogonal series for all independent variables in which. One of the positive aspects of this approach is that the use of orthogonal bases is well-grounded and easy to automate calculations.

The aim of the work is the spectral method construction in the bases of classical orthogonal polynomials for the solving of mathematical physics problems in the presence of the fractional derivatives, in particular, gas motion in the pipelines.

2. Formulation of the problem

Nonstationary gas motiom in horizontal pipelines is described by the system of partial differential equations which has the form [14]

$$\begin{cases} \dfrac{\partial \omega(x,t)}{\partial t} + \dfrac{\partial p(x,t)}{\partial x} + a\omega(x,t) - bp(x,t) = 0, \\ \dfrac{\partial \omega(x,t)}{\partial x} + \dfrac{1}{c^2}\dfrac{\partial p(x,t)}{\partial t} = 0, \end{cases} \tag{1}$$

here p, ω is the gas pressure and the mass velocity of gas motion accordingly; t is the time; x is the movable coordinate $x \in [0, L]$; L is the length of pipeline; $a = \upsilon_1 + \upsilon_2$, $b = -\frac{1}{4}(\upsilon_1^2 + \upsilon_2^2)$; υ_1 and υ_2 are the limits of change of gas motion velocity; c is the sound velocity in gas.

It is evident that to formulate the accordant problem of mathematical physics it is necessary to set the initial conditions and the limiting (boundary) conditions for the gas pressure or the volumetric gas consumption which are the desired functions. The boundary conditions for the desired functions are set depending on known input data.

In order to take better into account of the process history let replace the time derivative $\frac{\partial}{\partial t}$ with the fractional derivative in Riemann-Liouville terms [4, 10, 11, 15, 16]

$$D_t^\alpha = \frac{\partial^\alpha}{\partial t^\alpha}\varphi(t) := \frac{1}{\Gamma(\mu+1-\alpha)}\frac{\partial^{\mu+1}}{\partial \zeta^{\mu+1}}\int_0^t \frac{\varphi(\zeta)}{(t-\zeta)^{\alpha-\mu}}d\zeta, \tag{2}$$

there μ is the integer part of real number.

3. Research results

Since our system of equations containes the derivatives then $\mu = 0$ in this case. We obtain the following system of equations

$$\begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \dfrac{\partial}{\partial t} \displaystyle\int_0^t \dfrac{\omega(x,\zeta)}{(t-\zeta)^\alpha} d\zeta + \dfrac{\partial p}{\partial x} + a\omega - bp = 0, \\ \dfrac{\partial \omega}{\partial x} + \dfrac{1}{c^2} \dfrac{1}{\Gamma(1-\alpha)} \dfrac{\partial}{\partial t} \displaystyle\int_0^t \dfrac{p(x,\zeta)}{(t-\zeta)^\alpha} d\zeta = 0. \end{cases} \tag{3}$$

Let us expand the functions $p(x,t)$ and $\omega(x,t)$ which are included in the problem solution in Fourier series by Laguerre polynomials $L_m(t)$ [17]

$$p(x,t) = \sum_{m=0}^{\infty} p_m(x) L_m(t), \quad \omega(x,t) = \sum_{m=0}^{\infty} \omega_m(x) L_m(t), \tag{4}$$

here the coefficients $p_m(x)$, $\omega_m(x)$ are determined by the integral equations

$$p_m(x) = \int_0^\infty e^{-t} p(x,t) L_m(t) \quad \text{and} \quad \omega_m(x) = \int_0^\infty e^{-t} \omega(x,t) L_m(t). \tag{5}$$

Since [17]

$$\int_0^t L_n(t-\tau) L_m(\tau) d\tau = \int_0^t L_{n+m}(\tau) d\tau, \tag{6}$$

then, if we use the formula (6) and expand the function $k(t)$, which is the kernel of the integral equation (2), in Fourier series by Laguerre polynomials $L_m(t)$, we will get the equation

$$\frac{d}{dt} \int_0^t k(t-\tau) p(\tau) d\tau = \sum_{n=0}^{\infty} k_n \sum_{m=0}^{\infty} p_m L_{n+m}(t) = \sum_{n=0}^{\infty} c_n L_n(t), \tag{7}$$

there k_n and p_m are Fourier-Laguerre coefficients of the functions $k(t)$ and $p(t)$.

If we put the functions in the form of series by Laguerre polynomials in the initial system of equation (1) we will get the following system of equations

$$\begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \displaystyle\sum_{n=0}^{\infty} c_n(x) L_n(t) + \sum_{n=0}^{\infty} p_n'(x) L_n(t) + a \sum_{n=0}^{\infty} \omega_n(x) L_n(t) - b \sum_{n=0}^{\infty} p_n(x) L_n(t) = 0, \\ \displaystyle\sum_{n=0}^{\infty} \omega_n'(x) L_n(t) + \dfrac{1}{c^2} \dfrac{1}{\Gamma(1-\alpha)} \sum_{n=0}^{\infty} d_n(x) L_n(t) = 0. \end{cases} \tag{8}$$

In the latter formula

$$c_n(x) = \sum_{m=0}^{n} k_m \omega_{n-m}(x) = \sum_{m=0}^{n} \omega_m(x) k_{n-m}, \quad d_n(x) = \sum_{m=0}^{n} k_m p_{n-m}(x) = \sum_{m=0}^{n} p_m(x) k_{n-m}.$$

If we equate the coefficients at the same values $L_n(t)$ from the system (8) we will obtain the following system of ordinary differential equations for the determining of unknown coefficients

$$\begin{cases} \dfrac{1}{\Gamma(1-\alpha)} c_n(x) + p_n'(x) + a\omega_n(x) - bp_n(x) = 0, \\ \omega_n'(x) + \dfrac{1}{c^2} \dfrac{1}{\Gamma(1-\alpha)} d_n(x) = 0. \end{cases} \tag{9}$$

For $n = 0$ $c_0(x) = k_0\omega_0(x)$, a $d_0(x) = k_0 p_0(x)$. Thus for determining $p_0(x)$ and $\omega_0(x)$ we have the following system of ordinary differential equations

$$\begin{cases} \left(\dfrac{1}{\Gamma(1-\alpha)}k_0 + a\right)\omega_0(x) + p_0'(x) - bp_0(x) = 0, \\ \omega_0'(x) + \dfrac{1}{c^2}\dfrac{1}{\Gamma(1-\alpha)}k_0 p_0(x) = 0. \end{cases} \tag{10}$$

The solution of the latter system we will find in the form of exponential Fourier series of such type

$$p_n(x) = \sum_{j=-\infty}^{\infty} p_{nj}(\tau)\, e^{j\pi i x/l}, \quad x \in (0, l), \tag{11}$$

there

$$p_{nj}(\tau) = \frac{1}{l}\int_0^l p_j(x)\, e^{-n\pi i x/l}\, dx. \tag{12}$$

As a result of applying of the series (11) to the system (10) we obtain the following system of algebraic equations for the determining of the generalized spectra p_{0j} and ω_{0j}

$$\begin{cases} \dfrac{2}{l}\left(p_0(l)(-1)^j - p_0(0)\right) + \dfrac{j\pi i}{l}p_{0j} - bp_{0j} + \gamma_{\omega j}\omega_{0j} = 0, \\ \dfrac{2}{l}\left(\omega_0(l)(-1)^j - \omega_0(0)\right) + \dfrac{j\pi i}{l}\omega_{0j} + \gamma_{pj}p_{0j} = 0, \end{cases} \tag{13}$$

there

$$\gamma_{\omega 0j} = \frac{1}{\Gamma(1-\alpha)}k_0 + a, \quad \gamma_{p0j} = \frac{1}{c^2}\frac{k_0}{\Gamma(1-\alpha)}.$$

If we use the following notations

$$\alpha_{p0j} = -\frac{2}{l}\left(p_0(l)(-1)^j - p_0(0)\right), \ \beta_{p0j} = \frac{j\pi i}{l} - b, \ \alpha_{\omega 0j} = -\frac{2}{l}\left(\omega_0(l)(-1)^j - \omega_0(0)\right), \ \beta_{\omega 0j} = \frac{j\pi i}{l},$$

then the system (13) will look like

$$\begin{cases} \beta_{p0j}p_{0j} + \gamma_{\omega 0j}\omega_{0j} = \alpha_{p0j}, \\ \gamma_{p0j}p_{0j} + \beta_{\omega 0j}\omega_{0j} = \alpha_{w0j}. \end{cases} \tag{14}$$

The values of the coefficients $p_0(0)$, $p_0(l)$, $\omega_0(0)$, $\omega_0(l)$ are calculated by the presentation of boundary conditions in series by Laguerre polynomials.

$$p_0(x) = \int_0^\infty e^{-t}p(x,t)L_0(t)\, dt, \quad \omega_0(x) = \int_0^\infty e^{-t}\omega(x,t)L_0(t)\, dt. \tag{15}$$

The unknown values p_{0j} and ω_{0j} of the system (14) are calculated by the formulas

$$p_{0j} = \frac{\Delta_{p0j}}{\Delta_{0j}}, \quad \omega_{0j} = \frac{\Delta_{\omega 0j}}{\Delta_{0j}}, \tag{16}$$

where $\Delta_{0j} = \beta_{p0j}\beta_{\omega 0j} - \gamma_{\omega 0j}\gamma_{p0j}$, $\Delta_{p0j} = \alpha_{p0j}\beta_{\omega 0j} - \gamma_{\omega 0j}\alpha_{\omega 0j}$, $\Delta_{\omega 0j} = \alpha_{p0j}\gamma_{p0j} - \beta_{p0j}\alpha_{\omega 0j}$.

Then

$$p_0(x) = \sum_{j=-\infty}^{\infty} p_{0j}e^{-\frac{j\pi i x}{l}}. \tag{17}$$

The formulas for the determining of the coefficients p_{nj} and ω_{nj} for the arbitrary values n are obtained similarly

$$p_{nj} = \frac{\Delta_{pnj}}{\Delta_{nj}}, \quad \omega_{nj} = \frac{\Delta_{\omega nj}}{\Delta_{nj}},$$

where $\Delta_{nj} = \beta_{pnj}\beta_{\omega nj} - \gamma_{\omega nj}\gamma_{pnj}$,

$$\Delta_{pnj} = \alpha_{pnj}\beta_{\omega nj} - \alpha_{\omega nj}\gamma_{\omega nj} - \frac{\beta_{\omega nj}}{\Gamma(1-\alpha)}\sum_{i=1}^{n} k_i\omega_{n-i} + \frac{\gamma_{\omega nj}}{c^2\Gamma(1-\alpha)}\sum_{i=1}^{n} k_i p_{n-i},$$

$$\Delta_{\omega nj} = \alpha_{pnj}\gamma_{pnj} - \alpha_{\omega nj}\beta_{pnj} - \frac{\gamma_{pnj}}{\Gamma(1-\alpha)}\sum_{i=1}^{n} k_i\omega_{n-i} + \frac{\beta_{pnj}}{c^2\Gamma(1-\alpha)}\sum_{i=1}^{n} k_i p_{n-i}.$$

If the coefficients p_{nj} and ω_{nj} are found then the functions $p_n(x)$ and $\omega_n(x)$ are calculated by the formulas of the type (11) and the values of desired solutions are calculated by the formulas:

$$p(x,t) = \sum_{n=0}^{\infty}\sum_{j=-\infty}^{\infty} p_{jn}(\tau)\, e^{n\pi jx/l} L_n(t), \tag{18}$$

$$\omega(x,t) = \sum_{n=0}^{\infty}\sum_{j=-\infty}^{\infty} \omega_{jn}(\tau)\, e^{n\pi jx/l} L_n(t). \tag{19}$$

The numerical values of the coefficients $c_n(x)$ and $d_n(x)$ depend on the coefficients of the expansion of the integral equation kernel $k(t) = t^{-\alpha}$ in series by Laguerre polynomials which look like

$$k_m = \frac{\Gamma(m+\alpha)}{\Gamma(m+1)}\frac{\Gamma(1-\alpha)}{\Gamma(\alpha)}.$$

If $m \to \infty$, then

$$k_m = \frac{\Gamma(m+\alpha)}{\Gamma(m+1)}\frac{\Gamma(1-\alpha)}{\Gamma(\alpha)} \approx \frac{\Gamma(1-\alpha)}{\Gamma(\alpha)} m^{\alpha-1}.$$

In practical problems the fractional derivative parameter α is almost equal to one. Therefore the coefficients k_m will slowly approximate to zero. It, in turn, will lead to slow series (18) and (19) convergence. In this regard, it is expedient to use Chebyshev-Laguerre polynomials $L_n^{\lambda}(t)$ to find the solution of the formulated problem, there $\lambda > -1$ is the arbitrary parameter, and for functions approximation to use the representation of type [17]

$$k(t) = t^{\lambda}\sum_{m=0}^{\infty}\frac{k_m}{r_m} L_m^{\lambda}(t). \tag{20}$$

In such representation the series coefficients will look like

$$k_m = \int_0^{\infty} e^{-t} L_m^{\lambda}(t) k(t)dt, \tag{21}$$

and the normalizing multiplier r_m is calculated by the formula

$$r_m = \int_0^{\infty} e^{-x} L_m^{\lambda}(x) L_m^{\lambda}(x)dx = \frac{(1+\lambda)_m(\lambda)_m}{m!m!}\,{}_3F_2(-m,1,1-\lambda;\lambda+1,1-\lambda-m;1).$$

In our case $k(t) = t^{-\alpha}$ and generalized Fourier-Laguerre spectra for this function is calculated by the formula

$$k_m = \frac{\Gamma(m+\lambda+\alpha)\Gamma(1-\alpha)}{\Gamma(m+1)\Gamma(\lambda+\alpha)}. \tag{22}$$

Then for the large values m

$$k_m \approx \frac{\Gamma(1-\beta)}{\Gamma(\lambda+\beta)} m^{\lambda+\alpha-1}.$$

The latter formula gives the opportunity to evaluate the impact of the free parameter λ on the convergence velocity of the accordant series. However, the function representation by the series of type (20) has advantage in that the agreement of the choice of parameter λ with the behavior of the function $k(t)$ accelerates the rate of the series convergence. Let us submit the functions $k(t)$ and $p(x,t)$ in the form of Fourier series by the polynomials $L_n^{\lambda_k}(t)$, $\lambda_k > -1$, and $L_n^{\lambda_p}(t)$, $\lambda_p > -1$, accordingly. Since [17, 18]

$$\int_0^t (t-\tau)^{\lambda_k} L_m^{\lambda_k}(t-\tau)\tau^{\lambda_f} L_n^{\lambda_f}(\tau)\,d\tau = \frac{(n+m)!}{n!m!} B\left(\lambda_k+m+1, n+\lambda_f+1\right) t^{\lambda_k+\lambda_f+1} L_{n+m}^{\lambda_k+\lambda_f+1}(t),$$

Then the equation

$$\frac{\partial\omega}{\partial x} + \frac{1}{c^2}\frac{1}{\Gamma(1-\alpha)}\frac{\partial}{\partial t}\int_0^t \frac{p(x,\zeta)}{(t-\zeta)^\alpha} d\zeta = 0$$

will look like

$$\frac{\partial\omega}{\partial x} + \frac{1}{c^2}\frac{1}{\Gamma(1-\alpha)}\frac{\partial}{\partial t}\sum_{m=0}^{\infty}\frac{m!k_m}{\Gamma(m+\lambda_k+1)}\sum_{n=0}^{\infty}\frac{n!p_n(x)}{\Gamma(n+\lambda_p+1)} \times \frac{(n+m)!}{n!m!} B\left(\lambda_k+m+1, n+\lambda_p+1\right) t^{\lambda_k+\lambda_p+1} L_{n+m}^{\lambda_k+\lambda_p+1}(t) = 0,$$

or

$$\frac{\partial\omega}{\partial x} + \frac{1}{c^2}\frac{1}{\Gamma(1-\alpha)}\sum_{m=0}^{\infty}\frac{m!k_m}{\Gamma(m+\lambda_k+1)}\sum_{n=0}^{\infty}\frac{n!p_n(x)}{\Gamma(n+\lambda_f+1)} \times \frac{(n+m)!}{n!m!} B\left(\lambda_k+m+1, n+\lambda_p+1\right)\left(n+m+\lambda_k+\lambda_p+1\right)_1 t^{\lambda_k+\lambda_p} L_{n+m}^{\lambda_k+\lambda_p}(t) = 0.$$

If we regroup the additions in double sum of the right part of the latter formula we will obtain the equation

$$\frac{\partial\omega}{\partial x} + \frac{1}{c^2}\frac{1}{\Gamma(1-\alpha)} t^{\lambda_k+\lambda_p}\sum_{n=0}^{\infty} d_n(x) L_{n+m}^{\lambda_k+\lambda_p}(t) = 0.$$

In the latter formula

$$d_n(x) = \sum_{m=0}^{n} k_m p_{n-m}(x) = \sum_{m=0}^{n} k_{n-m} p_m(x).$$

If we note the mass consumption $\omega(x,t)$ as the series

$$\omega(x,t) = t^{\lambda_k+\lambda_p}\sum_{n=0}^{\infty}\frac{n!\,\omega_n(x)}{\Gamma(n+\lambda_p+1)} L_n^{\lambda_k+\lambda_p}(t),$$

we will obtain the following recurrent system of ordinary differential equations relatively the unknown coefficients $\omega_n(x)$ and $p_n(x)$

$$\frac{n!}{\Gamma(n+\lambda_p+1)}\frac{d\omega_n(x)}{dx}-\frac{1}{c^2}\frac{1}{\Gamma(1-\alpha)}d_n(x)=0. \tag{23}$$

A similar system is obtained from the first equation of the system (3)

$$\frac{1}{\Gamma(1-\alpha)}\frac{\partial}{\partial t}\int_0^t\frac{\omega(x,\zeta)}{(t-\zeta)^\alpha}d\zeta+\frac{\partial p}{\partial x}+a\omega-bp=0.$$

Applying similar expansions in the latter equation, we get

$$\frac{n!}{\Gamma(n+\lambda_p+1)}\frac{dp_n(x)}{dx}+\frac{1}{\Gamma(1-\alpha)}c_n(x)+a\omega_n(x)-bp_n(x)=0, \tag{24}$$

$$c_n(x)=\sum_{m=0}^{n}k_m\omega_{n-m}(x)=\sum_{m=0}^{n}k_{n-m}\omega_m(x).$$

The systems (23) and (24) are recurrent relatively the unknown generalized spectras. If we solve them we will find $\omega_n(x)$ and $p_n(x)$ for the arbitrary values n by the spectral method described above.

4. Discussion and conclusions

The proposed approach makes it possible to construct the effective algorithm for solving of differential equations or the systems of differential equations in the presence of the fractional time derivative. Since the properties of orthogonal polynomials are well-studied then it allows one to do a significant amount of calculations once and use them in subsequent cases. More, if the input data is set in a descrete form then similar to the paper [17] the algorithm can be submitted in matrix form. The efficiency of such approach is confirmed by the computational experiment. In Table 1 the results of calculations of the series (20) coefficients for different parameter λ values are presented.

Table 1. The results of calculations of the series (20) coefficients for different parameter λ values.

$\alpha = 0.9$			
n	k_n, $\lambda=-\alpha-\varepsilon$	k_n, $\lambda=-\alpha$	k_n, $\lambda=-\alpha+\varepsilon$
0	4.590417	4.590417	4.590417
1	−0.0459041725945783	0.000000	0.0459041725945783
2	−0.0227225654343163	0.000000	0.0231816071602621
3	−0.0150726350714298	0.000000	0.0155316767973756
4	−0.0112667947158938	0.000000	0.0116875867900251
5	−0.00899090218328323	0.000000	0.00937344460560015
6	−0.00747743364909722	0.000000	0.00782682624567612
7	−0.00639854679401319	0.000000	0.0067198893909305
8	−0.00559073026126903	0.000000	0.00588830307880285
9	−0.00496332608750439	0.000000	0.00524058974013454
10	−0.00446203015266645	0.000000	0.00472177135586122

From the results obtained it follows that if $\lambda=-\alpha$ then the coefficients of function $k(t)=t^{-\alpha}$ are equal to zero. That is for such a choice of the parameter λ we will have the following formulas

$$d_n(x)=k_0p_n(x), \quad c_n(x)=k_0\omega_n(x).$$

However, as can be seen from the formula (22) such an approach to choosing the parameter λ allows one to eccelerate the convergence of accordant Fourier-Laguerre series.

It is necessary to notice that the summing of Fourier-Laguerre series is sensitive to parameter λ selecting. From the results obtained we can conclude that the sensitivity of Furier-Laguerre series summing to the choice of parameter and the need of addititonal researches in summing operations of these series are confirmed whereas Chebyshev-Laguerre polynomials have significant disadvantage is that for the large n their behavior is following

$$L_n^\lambda(t) = O\left(e^{t/2} t^{-(2\lambda+1)/4} n^{(2\lambda-1)/4}\right).$$

This property of the polynomials considerably narrows the class of problems Chebyshev-Laguerre polynomials are used in which because there are the computational difficulties during the series summing for the large values t. In practice this problem is solved by the introduction of the scaling multiplier. However, the change of the scaling multiplier requires redefining the problem and leads to instability in calculation of the desired function. Therefore, the Chebyshev-Laguerre transform is generalized as follows.

Introduce the integral transform

$$f_n = \int_0^\infty t^{\nu\lambda+\nu-1} e^{-\mu t^\nu} L_n^\lambda(\mu t^\nu) f(t)\, dt,$$

there $n = 0, 1, 2, \ldots$, $\mu > 0$, $|\nu| < \infty$, $\nu \neq 0$. Then the formula of the reverse will look like

$$f(t) = \sum_{n=0}^{\infty} \frac{n! f_n}{\Gamma(n+\lambda+1)} L_n^\lambda(\mu t^\nu).$$

Choosing of free parameters μ and ν allows us to construct the regularizing algorithm for calculating of Fourier-Laguerre coefficients f_n and summing of accordant orthogonal series.

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Застосування ортогональних многочленів для розв'язування систем диференціальних рівнянь за наявності похідних дробового порядку

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Побудовано математичну модель руху газу в трубопроводах для випадку, коли неусталений процес описано похідною дробового порядку за часовою змінною. Сформульовано крайову задачу. Рішення задачі знаходять спектральним методом в базисах многочленів Чебишева–Лагерра за часовою змінною та многочленів Лежандра за координатою. Знаходження рішення в результаті зведено до системи алгебраїчних рівнянь. Проведено числовий експеримент.

Ключові слова: *математична модель, рух газу в трубопроводах, спектральні методи, ортогональні многочлени.*

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