UDC 517.96:004

Gorak M.., Dyubo E.
Information Technologies and Systems Department,
Lugansk Taras Shevchenko
National Pedagogical University
Oboronnaya Str., 2a, Lugansk, 91011, Ukraine,
E-mail: Marina_Gorak@rambler.ru

SOME ASPECTS OF SOLVING THE INTEGRATED EQUATIONS WITH USE OF THE MODERN SOFTWARE

© Gorak M., Dyubo E.

The article is devoted to the consideration of numerical methods of the decision of Fredgolmans' integrated equations of the second sort in the language of Fortran and Pascal and its realization in the applied mathematical package MathCad.

Keywords - integrated equation, Fredgolmans' linear integrated equation of the second sort.

Usually, the problem of solving the integrated equations arises as auxiliary form which helps to solve boundary tasks for the differential equations with individual derivatives and as independent when we research the work of the nuclear reactors, solve the so-called return tasks in geophysics or process the results of supervision, in mathematical economy, etc.

The purpose of the research is to consider numerical methods which help to solve the integrated equations and to look through the ways of their realization with the use of programming languages and modern mathematical packages. We will lead the comparative characteristic of the indicated ways of their realization. Fredgolmans' linear integrated equations of the second sort have been chosen as an object of research.

We should remind that Fredgolmans' linear integrated equation of the second sort is called the equation (Eq. 1):

$$y - \lambda Gy = y - \lambda \int_a^b K(x,s)y(s)ds = f(x), \qquad (1)$$

where y - is unknown function;

$K(x,s)$ and $f(x)$ -are known functions;

x and s - are the valid variables changing in an interval (a,b) ;

λ -is a numerical multiplier.

Function $K(x,s)$ is a nucleus of the integrated Eq. (1). It is supposed, that the nucleus $K(x,s)$ is determined in a square $\Omega\{a \le x \le b, a \le s \le b\}$ on a plane (x,s) and continues in Ω . It has such breaks that double integral has the final value

$$\int_a^b \int_a^b |K(x,s)|^2 dxds .$$

Will use some formula of numerical integration

$$J(\psi) = \int_a^b \psi(x)dx \approx S_m(\psi) = \sum_{j=1}^m c_j \psi(x_j^{(m)}), \qquad (2)$$

where c_j depends on m . Then we have the equality

$$J(\psi) = S_m(\psi) + R_m(\psi), \qquad (3)$$

where $R_m(\psi)$ - is a residual member of quadrature Eq. (2).

Applying Eq. (3) to Fredgolmans' equation of the second sort (Eq. 1), we'll take:

$$y(x) - \lambda \sum_{j=1}^m c_j K(x, x_j^{(m)}) y(x_j^{(m)}) - R_m(\lambda Ky) = f(x). \qquad (4)$$

The residual member $R_m(\lambda Ky)$ during the integral calculation $\lambda \int_a^b K(x,s)y(s)ds$ with the help of a quadrature Eq. (2) - is a function of a variable x. Supposing that in Eq. (4) $x = x_i^{(m)}$, $i = \overline{1,m}$ we'll take the system of the equations:

$$y(x_i^{(m)}) - \lambda \sum_{j=1}^m c_k K(x_i^{(m)}, x_j^{(m)}) y(x_j^{(m)}) = f(x_i^{(m)}) + R_m(\lambda Ky)\big|_{x_i^{(m)}}.$$

If we reject a residual member, we get a system of the linear algebraic equations.

$$y_i - \lambda \sum_{j=1}^m c_j K(x_i^{(m)}, x_j^{(m)}) y_j = f_j, \ f_i = f(x_i^{(m)}), \ i = \overline{1,m}. \qquad (5)$$

If we want to solve the system of the linear Eqs. (5), the standard methods of solving systems of the linear algebraic equations can be used.

In system Eqs. (5) sizes y_1, y_2, …, y_m are the approximate values for solving y Eqs. (1) in units x_1, x_2, …, x_m, where

$$x_1 = a, \ x_2 = a+h, \ ..., \ x_m = a+(m-1)h, \ h = \frac{b-a}{n-1}. \qquad (6)$$

Factors' values c_m according to Simpson's formula are ($m = 2l+1$):

$$c_1 = c_{2l+1} = \frac{h}{3}, \quad c_2 = c_4 = ... = c_{2l} = \frac{4h}{3}, \quad c_3 = c_5 = ... = c_{2l-1} = \frac{2h}{3}. \qquad (7)$$

Using this algorithm, it is possible to write the program with the help of one language among numerous programming languages to take a great amount of solutions for the Eq. (1). Here are some subprograms in languages Pascal and the FORTRAN. The given languages remain the most used in the scientific environment.

Designations which are used:

M - the quantity of central points;

N=M+1;

D= λ;

X - is one-dimensional file of M size which contains central points x_k, $k = \overline{1,m}$;

C - is one-dimensional file of M size which contains values of quadrature factors in formulas like c_k, $k = \overline{1,m}$;

Ker - is a bidimentional file of (M, M) size, which contains the values $K(x_i^{(m)}, x_j^{(m)})$, $1 \le i, j \le M$, where i - is a number of a line, and j - is a number of a file column;

F - is one-dimensional file of M size which contains values of the right parts of the Eq. (1);

R - is one-dimensional file of M size which, after the reference to the subroutine, will contain the solutions of the Eq. (1).

We propose next texts of subprograms FredgEq in the FORTRAN and Pascal languages (Table 1). The subprogram GaussEq is used in these subprograms. It's also used for solving the system of the linear algebraic equations with the help of Gauss method.

TABLE 1

TEXTS OF SUBPROGRAMS FREDGEQ IN THE FORTRAN AND PASCAL LANGUAGES

Fortran	Pascal

SUBROUTINE FREDGEQ (M,N,D,X,C,KER,F,R)
  DIMENSION X(M), C(M), KER(M,M), F(M) ,R(M),
 A(M,N)
DO 10 I=1,M
DO 5 J=1,M
5 A(I,J)= –D*C(J)*KER(I,J)
10 A(I,J)=A(I,I)+1
  DO 15 I=1,M
15 A(I,N)=F(I)
  CALL GAUSSEQ (A,R)
  RETURN
  END
	Procedure FredgEq (M:integer; D:real; C:ar2; KER:ar4;
	F:ar2; A:ar1;E:real;Var R:ar2);
	 var
	  i,j:integer;
	 begin
	  for i:=1 to M do
	  begin
	   for j:=1 to M do
	   A[i,j]:=-D*C[j]*KER[i,j];
	   A[i,i]:=A[i,i]+1;
	  end;
	  for i:=1 to M do
	  begin
	   A[i,M+1]:=F[i];
	  end;
	  for i:=1 to M do
	  begin
	   for j:=1 to M+1 do
	   begin
	    write(A[i,j]:5:3,' ');
	   end;
	   writeln;
	  end;
	  GaussEq (A,M,R,E);
	 end;

Such type of programs can be used today. But there is a set of applied mathematical packages which allow to solve mathematical and engineering problems without deepening in the specificity of programming language. One of such programs is a program MathCad which can be named modern, universal and mass mathematical system. It allows to carry out both numerical and symbolical calculations; it has the convenient mathematic-oriented interface. The system contains a set of functions which realize algorithms of numerical methods. As an example we will consider the decision of the integrated Fredgolmans' equation in the system of MathCad:

$$y(x) - \lambda \int_0^\pi s\cos^2 xy(s)ds = \sin x - \frac{\pi}{4}(4-\pi)\cos^2 x - 0,5 .$$

By applying of Simpson's formula and choosing units in points

$$x_k = \frac{\pi k}{16}, \ k = \overline{0,16}$$

we will generate matrixes X and A which will contain accordingly central points x_k of Eq. (6) and coefficients of quadrature formulas c_k of Eq. (7), $k = \overline{0,16}$.

$$n := 17$$
$$k := 0, 1 .. n - 1$$
$$X_k := \frac{\pi \cdot k}{16}$$
$$h := \frac{\pi}{16}$$

$$A(h) := \begin{vmatrix} A_0 \leftarrow \dfrac{h}{3} \\ i \leftarrow 1 \\ \text{while } i \leq n - 1 \\ \quad \begin{vmatrix} A_i \leftarrow 4 \cdot \dfrac{h}{3} \\ A_{i+1} \leftarrow 2 \cdot \dfrac{h}{3} \\ i \leftarrow i + 2 \end{vmatrix} \\ A_{n-1} \leftarrow A_0 \\ A \end{vmatrix}$$

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Will generate matrix F which will contain values of the right parts of the equation:

$$f(x) := \sin(x) - \frac{\pi}{4} \cdot (4 - \pi) \cdot (\cos(x))^2 - 0.5$$

$$F_k := f(X_k)$$

Will generate matrix K - a bidimentional file of the size (17, 17), which contains values $K(x_i^{(m)}, x_j^{(m)})$, $0 \le i, j \le 16$, where i - number of the line and j - number of the column of a file.

$$K(X) := \begin{vmatrix} \text{for } i \in 0..n-1 \\ \quad \text{for } j \in 0..n-1 \\ \qquad K_{i,j} \leftarrow X_j \cdot (\cos(X_i))^2 \\ K \end{vmatrix}$$

Will generate file Koeff which will contain factors of system of the linear Eqs. (5) for the considered equation:

$$Koeff(C, A, K1) := \begin{vmatrix} \text{for } i \in 0..n-1 \\ \quad \text{for } j \in 0..n-1 \\ \qquad Koeff_{i,j} \leftarrow -C \cdot A_j + K1_{i,j} \\ \quad Koeff_{i,i} \leftarrow Koeff_{i,i} + 1 \\ Koeff \end{vmatrix}$$

Will generate a matrix in which will contain the expanded matrix of the system of the linear Eqs. (5) for the considered equation:

$$B := \text{augment}(Koeff1, F)$$

After will used Gauss method for the decision of system of the linear equations by using of the built - in function rref:

$$C1 := \text{rref}(B)$$

Results will be received:

$X1^T = $

	0	1	2	3	4	5	6	7	8
0	-0.825	-0.612	-0.374	-0.125	0.116	0.33	0.498	0.606	0.643

Thus there are a lot of methods of the realization of numerical methods of the decision of the integrated equations. Existing programming languages give the user huge opportunities from the point of view of their applicability for the decision of various types of problems. But from the point of user's view who cannot and does not want to study the programming language, the best variant will be the using of the specially applied software.

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