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# RESEARCH ON EFFICIENCY OF TELEPHONE NETWORKS WITH DIFFERENT STRUCTURES 

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#### Abstract

With the development of communication networks, their structures have also evolved and improved. Today the prospective development and construction of telephone networks of all levels of hierarchy calls for insight into their structures. The aim of this paper is determination and analysis of the efficiency of the most widespread structures of public telephone networks according to specific criteria for effectiveness. The paper sets the following tasks: 1) to analyze the most common structures of telephone networks and to build them on a particular area taking into account the initial data and methods for nodes locating; 2) to establish parameters of the telephone networks efficiency; 3) to calculate the chosen parameters for the building of structures and conduct a quantitative and qualitative analysis of the results obtained. The relevancy of this work is explained both by the lack, in scientific literature, of quantitative research into the parameters of efficiency of telephone networks with different structures and by practical needs related to the common use of various structures of the communication networks.


Key words: telephone network with channel switching, digital switching system, network structure, reliability, bandwidth.

## 1. Introduction

A telephone communication network is formed by switching nodes (telephone exchanges), data terminal devices, connecting channels and communications lines [1].The networks can have different structures, i.e. differ in number and location of nodes, as well as in character of their interrelation. There are different types of Public Switched Telephone Networks (PSTN) [2], namely: international, national (inter-city), zonal, municipal and rural. Networks of the last two types share a common name - local area telephone networks (LTN). According to [1-3], the main costs assigned to building a network are spent on line structures and therefore the problem of choosing a network structure from the perspective of the highest efficiency level is so pressing.

The aim of this paper is determination and analysis of the efficiency of the most widespread structures of
public telephone networks according to specific criteria for effectiveness.

## 2. Choosing telephone network structures to be

 studied in the terms of efficiencyOn analysing the PSTN, it should be noted, that such main structures as fully-connected and radial-nodal are the basis for the construction of international, national and zonal telephone networks. And such structures as linear, nodal, star-like and circular are found only in certain types of PSTN, in particular, local with incoming message nodes (LTN with IMN), local with incoming and outgoing message nodes (LTN with IMN and OMN), non-zoned local, and rural [2, 4]. To do research, we chose seven structures that are appropriate for the building of PSTN of different hierarchy levels, namely: fully-connected, star-like, linear, radial-nodal structures, a network structure with IMN, IMN and OMN, as well as a circular one. For these structures to be built, there were set eight digital switching systems (DSS) that are located on the territory with a population of 1 million habitants. The determination of one DSS's capacity was provided by the standard for Ukraine density of telephone devices in accordance with ITU - T recommendations taking into account the coefficient of long-term development of the PSTN. Then the capacity of one DSS is 25000 subscriber lines (DL). The dimension of the territory on which the DSS is located has been chosen among the typical areas of the cities with the same population ( 100 square kilometers). To ease the task, we have chosen the territory that can be placed on an A4 sheet of paper and has the following dimensions $A=8 \mathrm{~km} \times 12 \mathrm{~km}=96 \mathrm{sq} . \mathrm{km}$. Depending on the initial data, the structure of the network can be selected taking into account a given location of nodes or a given fixed territory [3]. The former structure is called a graph network model, the latter - a territory-oriented network. In the work, the fixed territory that is shown in Fig. 1 is chosen in the scale of $1: 50000$. The geometrical principle of the network structure optimization is applied to allocate the nodes on this particular territory [3]. According to this principle, the entire territory $A$ is divided into equidimensional elements of the surface $F$ represented by rectilinear polygons that gives the possibility to fully cover this territory with the
elements. In the paper, we accept the quadrangular form of the elements $F$, with the quantity of the elements $F$ being equal to the quantity of nodes, i.e. 8. The sizes of the element F sides (legs) are derived from Fig. 1, and the value of $A_{F}: l_{1}=3 \mathrm{~km}$ and $l_{2}=4 \mathrm{~km}$. In order to determine the DSS location in the element F, we apply the arithmetic method [3]. This method is to provide the equal number of information sources on the left and on the right, from above and from below relative to the node set. The even location of information sources is accepted as a condition. It is also specified, that all these eight DSS with the accepted numeration are located in the centers of the elements $F$ (Fig. 1). For further research, the calculated distances between telephone exchanges are tabulated in Table 1.


Fig. 1. Layout of the digital switching systems.
Table 1
Distance between DSS, km

| № DSS | $\mathbf{3 1}$ | $\mathbf{3 2}$ | $\mathbf{3 3}$ | $\mathbf{3 4}$ | $\mathbf{4 1}$ | $\mathbf{4 2}$ | $\mathbf{4 3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{3 1}$ | 0 | 3.0 | 5.0 | 4.0 | 9.98 | 7.21 | 6.0 |
| $\mathbf{3 2}$ | 3.0 | 0 | 4.0 | 5.0 | 7.21 | 5.0 | 3.0 |
| $\mathbf{3 3}$ | 5.0 | 4.0 | 0 | 3.0 | 6.0 | 3.0 | 5.0 |
| $\mathbf{3 4}$ | 4.0 | 5.0 | 3.0 | 0 | 9.0 | 6.0 | 7.21 |
| $\mathbf{4 1}$ | 9.98 | 7.21 | 6.0 | 9.0 | 0 | 3.0 | 5.0 |
| $\mathbf{4 2}$ | 7.21 | 5.0 | 3.0 | 6.0 | 3.0 | 0 | 4.0 |
| $\mathbf{4 3}$ | 6.0 | 3.0 | 5.0 | 7.21 | 5.0 | 4.0 | 0 |

## 3. Parameters of telephone network efficiency

The research into the efficiency of telephone networks with the above-enumerated structures was done on the basis of economic indicators and parameters of reliability. The economic indicators include:

1) A bandwidth in channels of one kilometer of the network $E$, chan./kilometer (introduced by the authors of the paper). This parameter directly characterizes the network bandwidth, that is measured by the number of channels $V$ per kilometer of the whole network length $L$ :

$$
\begin{equation*}
E=\frac{V}{L}, \text { chan. } / \mathrm{km} . \tag{1}
\end{equation*}
$$

The whole network length $L$ is determined as the sum of lengths of all lines (arcs) of the network, and depends on the structure of the network being studied. To determine the number of channels $V$, it is necessary to calculate load intensities in every node in the network, interexchange load, and at specified call losses define the number of channels in every arc. The total number of channels in the network is determined as the sum of channels of all network lines (arcs).

The calculation of the number of channels $V$ in the arcs required for the transmission of load from the DSS is conducted by employing the first Erlang formula at the call losses $p=0.005$ [4]:

$$
\begin{equation*}
p=E_{V}(Y)=\frac{Y^{V} / V!}{\sum_{i=0}^{V}\left(Y^{i} / i!\right)} \tag{2}
\end{equation*}
$$

where $E_{V}(Y)$ are the losses of the fully accessible non-blocking beam with lines $V$, that is supplied by the load $Y$ from a simpler stream of calls. In this work, the direction of the streams is chosen as initial (output).
2) a skipped intensity of the load at the set losses $p\left(Y_{p(M)}\right)$ [2]:

$$
\begin{equation*}
Y_{p}(M)=Y(1-p) \tag{3}
\end{equation*}
$$

where $Y$ is the intensity of the load that enters the network and equals the sum of load intensities in all network lines (arcs).
3) total cost of the network $C$ is determined as:

$$
\begin{equation*}
C=\gamma \cdot \sum_{b_{l m} \in B} l_{l m} \cdot V_{l m}^{\delta} \tag{4}
\end{equation*}
$$

where $\gamma$ is the coefficient that characterizes the cost of one kilometre of a channel. To make a comparative
analysis of the networks cost simple, we accept $\gamma=1 / \mathrm{km}$ chan. In this case, the cost of networks is determined in relative units. It is known that with the capacity (number of channels $V$ ) of channel-forming equipment increasing, its cost $Q$ grows according to the exponential law in relation to its capacity:

$$
\begin{equation*}
Q=\chi \cdot V^{\delta} \tag{5}
\end{equation*}
$$

where $\chi$ is the coefficient that characterizes the cost of one channel.

Fig. 2 depicts the dependences of increasing the relative cost of channel-forming equipment on its capacity at $\chi=1$ and different indicators $\delta(\delta=1 / 2,2 / 3,3 / 4)$. The dependence of $Q$ on the capacity when $\delta=2 / 3$ that corresponds to the real practical dependences is chosen as exponential. Then, the calculation of the network cost is conducted in accordance with (4) at $\delta=2 / 3$.


Fig. 2. The dependences of increase in relative cost of channel-forming equipment on its capacity (number of channels $V$ ).

Network reliability is its property to provide communication at certain exploitation conditions [1,5].

In the paper, the reliability of connection between any two nodes of the network represents the parameters, which characterize the property of network reliability [1, 3, 4]. When evaluating the reliability of connection between two DSS of the investigated networks, the parameters $\rho_{i j}^{(k)}$ and $\pi_{i j}^{(l)}$ are chosen. The reliability $\rho_{i j}^{(k)}$ of the k-th way $\mu_{i j}^{(k)}$ is the probability of operable state of all the arcs that form this way, i.e.

$$
\begin{align*}
& \rho_{i j}^{(k)}=\rho\left(\mu_{i j}^{(k)}\right)= \\
& \prod_{b_{l m} \in \mu_{i j}^{(k)}} p_{l m}=\prod_{b_{l m} \in \mu_{i j}^{(k)}} 1-q_{l m}, \tag{6}
\end{align*}
$$

where $\pi_{i j}^{(l)}$ is the probability that in the $l$-th cut $\sigma_{i j}^{(l)}$ there is at least one operable arc, which is determined from the ratio below:

$$
\begin{align*}
& \pi_{i j}^{(l)}=\pi\left(\sigma_{i j}^{(l)}\right)= \\
& 1-\prod_{b_{m p} \in \sigma_{i j}^{(l)}}\left(1-p_{m p}\right)  \tag{7}\\
& =1-\prod_{b_{m p} \in \sigma_{i j}^{(l)}} q_{m p}
\end{align*}
$$

In the formulae (6), (7), for the determination of $\rho_{i j}^{(k)}$ and $\quad \pi_{i j}^{(l)}: \quad p_{l m}=1-q_{l m} \quad$ denotes the probability of operable state of every arc $b_{i m}$ of the network, $p_{m p}=1-q_{m p}$ stands for the probability of operable state of every arc in the $l$ - th cut $\sigma_{i j}^{(l)}$. When determining these parameters, the most often used values of $p_{l m}=p=0.9$ are taken. When studying different structures of the network, we took into account that in some of them, the ways between the DSS have different number of arcs and different cuts that cause different reliability of connections between the nodes. In this case, the maximum ( $\pi_{\text {max }}$ ) and minimum ( $\pi_{\min }$ ) reliabilities of internode connections were calculated.

The results of the calculations concerning the intensity of interexchange loads and the number of channels between the DSSs are tabulated in Tables 2 and 3 respectively.

Table 2
Intensity of interexchange loads, Erl

| $\mathbf{N} \mathbf{N o}$ <br> DSS | $\mathbf{3 1}$ | $\mathbf{3 2}$ | $\mathbf{3 3}$ | $\mathbf{3 4}$ | $\mathbf{4 1}$ | $\mathbf{4 2}$ | $\mathbf{4 3}$ | $\mathbf{4 4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{3 1}$ | 190.78 | 141.17 | 118.28 | 129.73 | 83.94 | 101.11 | 110.65 | 89.666 |
| $\mathbf{3 2}$ | 129.64 | 175.2 | 119.13 | 108.62 | 92.856 | 108.62 | 129.64 | 101.61 |
| $\mathbf{3 3}$ | 108.62 | 119.13 | 175.2 | 129.64 | 101.61 | 129.64 | 108.62 | 92.856 |
| $\mathbf{3 4}$ | 129.73 | 118.28 | 141.17 | 190.78 | 89.666 | 110.65 | 101.11 | 83.94 |
| $\mathbf{4 1}$ | 83.94 | 101.11 | 110.65 | 89.666 | 190.78 | 141.17 | 118.28 | 129.73 |
| $\mathbf{4 2}$ | 92.856 | 108.62 | 129.64 | 101.61 | 129.64 | 175.2 | 119.13 | 108.62 |
| $\mathbf{4 3}$ | 101.61 | 129.64 | 108.62 | 92.856 | 108.62 | 119.13 | 175.2 | 129.64 |
| $\mathbf{4 4}$ | 89.666 | 110.65 | 101.11 | 83.94 | 129.73 | 118.28 | 141.17 | 190.78 |

Table 3

## Number of channels between DSS

| $\mathbf{N o}$ <br> DSS | $\mathbf{3 1}$ | $\mathbf{3 2}$ | $\mathbf{3 3}$ | $\mathbf{3 4}$ | $\mathbf{4 1}$ | $\mathbf{4 2}$ | $\mathbf{4 3}$ | $\mathbf{4 4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{3 1}$ | 218 | 166 | 141 | 154 | 104 | 123 | 133 | 110 |
| $\mathbf{3 2}$ | 154 | 202 | 142 | 131 | 114 | 131 | 154 | 123 |
| $\mathbf{3 3}$ | 131 | 142 | 202 | 154 | 123 | 154 | 131 | 114 |
| $\mathbf{3 4}$ | 154 | 141 | 166 | 218 | 110 | 133 | 123 | 104 |
| $\mathbf{4 1}$ | 104 | 123 | 133 | 110 | 218 | 166 | 141 | 154 |
| $\mathbf{4 2}$ | 114 | 131 | 154 | 123 | 154 | 202 | 142 | 131 |
| $\mathbf{4 3}$ | 123 | 154 | 131 | 114 | 131 | 142 | 202 | 154 |
| $\mathbf{4 4}$ | 110 | 133 | 123 | 104 | 154 | 141 | 166 | 218 |

## 4. Research into efficiency of telephone networks with different structures

Fig. 3a displays a fully-connected structure of the network, in which between any pair of nodes there exists a direct connection. The number of arcs $B$ equals
$N \cdot(N-1) / 2$, where $N$ is the number of DSS in the network, thus the network has 28 arcs. According to the dependences (1)-(7), the parameters of the fullyconnected network are calculated by using Tables 1, 2, 3, the data of which coincide with the lengths of the arcs (Table1), loads of the arcs (Table 2) and the number of channels in the arcs (Table 3) of the given network:

1) the total length of network arcs: $L=309.6 \mathrm{~km}$;
2) the total number of channels $V$ is 7520 channels;
3) the network bandwidth in chan $/ \mathrm{km}$ :

$$
E=\frac{V}{L}=\frac{7520}{309,6}=24,289=25 \text { chan } / \mathrm{km}
$$

4) the network bandwidth in Erlangs:

$$
\begin{aligned}
& Y_{p(M)}=Y(1-p)=6258.8 \cdot(1-0.005) \\
& =6227.52 \mathrm{Erl}
\end{aligned}
$$

5) the costs of all the arcs $C_{l m}$ in relative quantities, the values of which are represented in Table 4:
6) the relative cost of the network: $C=7812.8$;
7) the parameters of reliability:

$$
\rho=1-q=1-0.1=0.9
$$

$$
\pi=1-q^{7}=1-0.1^{7}=0.9999999
$$

In the star-like structure of the network (Fig. 3b), there is only one-way link between any two DSS, which consists of two arcs and a central node (CN). In accordance with dependences (1) - (7), the parameters for this type of network structure have been calculated:

1) the ranges of the arcs (Table 5);
2) the total range of the network arcs $L=30 \mathrm{~km}$;
3) the intensities of the arc load (Table 6);
4) the intensities of the network load $Y=12517.44 \mathrm{Erl} ;$
5) the number of the arc channels (Table 7);
6) the total number of the network channels $V=6624$ chan;
7) the network bandwidth $E$ in chan $/ \mathrm{km}$

$$
E=\frac{V}{L}=\frac{6624}{30}=220.8=221 \mathrm{chan} / \mathrm{km}
$$

8) the costs of all arcs $C_{l m}$ in relative quantities (Table 8);
9) the relative network cost as the sum of all arcs costs: $C=262.77$;
10) the parameters of reliability:

$$
\begin{gathered}
\rho=(1-q) \cdot(1-q)=(1-0.1) \cdot(1-0.1)=0.81 \\
\pi=1-q=1-0.1=0.9
\end{gathered}
$$

Costs of arcs $C_{l m}$ in a fully-connected network

| m | 31 | 32 | 33 | 34 | 41 | 42 | 43 | 44 | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 31 | - | 90.305 | 135 | 114.53 | 220.026 | 177.751 | 155.825 | 205.973 | 1098.8 |
| 32 | 85.903 | - | 108.51 | 128.55 | 168.979 | 128.55 | 85.903 | 147.92 | 854.31 |
| 33 | 128.55 | 108.51 | - | 85.903 | 147.92 | 85.903 | 128.55 | 168.97 | 854.31 |
| 34 | 114.53 | 135 | 90.305 | - | 205.973 | 155.92 | 177.75 | 220.02 | 1098.8 |
| 41 | 220.02 | 177.75 | 155.82 | 205.97 | - | 90.305 | 135 | 114 | 1098.8 |
| 42 | 168.97 | 128.55 | 85.903 | 147.92 | 85.903 | - | 108.51 | 128.55 | 854.31 |
| 43 | 147.92 | 85.903 | 128.55 | 168.97 | 128.55 | 108.51 | - | 85.903 | 854.31 |
| 44 | 205.97 | 155.82 | 175.75 | 220.02 | 114 | 135 | 90.305 | - | 1098.8 |


a) fully-connected

b) star-like

Fig. 3. Public Switched Telephone Network structures.

Table 6

Lengths of arcs $l_{l m}$ in a star-shaped network, km

| $\operatorname{blm}$ | $(31-\mathrm{CN})$ | $(32-\mathrm{CN})$ | $(33-\mathrm{CN})$ | $(34-\mathrm{CN})$ | $(41-\mathrm{CN})$ | $(42-\mathrm{CN})$ | $(43-\mathrm{CN})$ | $(44-\mathrm{CN})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{llm}$ | 5 | 2.5 | 2.5 | 5 | 5 | 2.5 | 2.5 | 5 |

## Arcs load $Y_{l m}$ in a star-like network, Erl

| $\mathrm{b}_{\operatorname{lm}}$ | $(31-\mathrm{CN})$ | $(32-\mathrm{CN})$ | $(33-\mathrm{CN})$ | $(34-\mathrm{CN})$ | $(41-\mathrm{CN})$ | $(42-\mathrm{CN})$ | $(43-\mathrm{CN})$ | $(44-\mathrm{CN})$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}_{\operatorname{lm}}$ | 774.55 | 790.13 | 790.13 | 774.55 | 774.55 | 790.13 | 790.13 | 774.55 |

Table 7
Number of channels in arcs $V_{l m}$ of a star-like network, chan

| $\mathrm{b}_{\mathrm{lm}}$ | $(31-\mathrm{CN})$ | $(32-\mathrm{CN})$ | $(33-\mathrm{CN})$ | $(34-\mathrm{CN})$ | $(41-\mathrm{CN})$ | $(42-\mathrm{CN})$ | $(43-\mathrm{CN})$ | $(44-\mathrm{CN})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{V}_{\mathrm{lm}}$ | 820 | 836 | 836 | 820 | 820 | 836 | 836 | 820 |

Table 8
Costs of arcs $C_{l m}$ of a star-like network

| blm | $(31-\mathrm{CN})$ | $(32-\mathrm{CN})$ | $(33-\mathrm{CN})$ | $(34-\mathrm{CN})$ | $(41-\mathrm{CN})$ | $(42-\mathrm{CN})$ | $(43-\mathrm{CN})$ | $(44-\mathrm{CN})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| clm | 435.08 | 220.86 | 220.86 | 436.08 | 436.08 | 220.86 | 220.86 | 436.08 |

Fig. 3c represents a network with a linear structure, in which there is only one-way connction between any two DSSs. The network consists of node zones (NZ 3) and ( NZ 4 ), and the connection between the DSS of various zones is arranged through a CN. For this network, we have calculated:

1) the lengths of the arcs NZ 3 and NZ 4: $L_{3}=$ $=L_{4}=13.5 \mathrm{~km}$, and the total lenghth of the network arcs:

$$
L=L_{3}+L_{4}=13.5+13.5=27 \mathrm{~km}
$$

2) the number of channels in the arcs of each zone and the total number of channels in the network:

$$
V=V_{3}+V_{4}=5890+5890=11780 \text { channels }
$$

3) the network bandwidth $E$ in chan $/ \mathrm{km}$ :

$$
E=\frac{V}{L}=\frac{11780}{27}=436.29=437 \text { chan } / \mathrm{km} ;
$$

4) the costs of each zone arcs and the cost of the network as the sum of costs of all the arcs: $C=2627.77$;
5) the parameters of reliability:

$$
\begin{gathered}
\rho_{\max }=1-q=1-0.1=0.9 ; \\
\rho_{(31-C N)}=\rho_{(41-C N)}=(1-q)^{4}=(1-0.1)^{4}=0.656 ; \\
\rho_{\min }=\rho_{(31-41)}=(1-q)^{7}=(1-0.1)^{7}=0.478 ; \\
\pi=1-q=1-0.1=0.9 .
\end{gathered}
$$

Fig. 3d illustrates a network with a radial-nodal structure, which is a sort of the tree-like structure with one-way link between any two DSSs. In accordance with dependences (1)-(7), the parameters for this type of network structure have been calculated (Table 9). Fig. 3g represents a local telephone network with the incoming message nodes (IMN), in which the entire territory of the city is divided into nodal areas (NA), and all the DSS of the same NA are connected according to the principle "each with each" and the connections of subscribers of different NA are established through the IMN [5]. For this network, we have calculated:

1) the lengths of the arcs of each NA (3 and 4): $L_{3}=L_{4}=54.1 \mathrm{~km}$, and the total arcs lengths of the network as a whole:

$$
L=L_{3}+L_{4}=54.1+54.1=108.2 \mathrm{~km} ;
$$

2) the number of channels in the arcs of each NA: $V_{3}=V_{4}=5340$ channels, and the total number of channels in the network: $V=V_{3}+V_{4}=5340+5340=10680$ channels;
3) the network bandwidth in chan $/ \mathrm{km}$ :

$$
E=\frac{V}{L}=\frac{10680}{108.2}=98.7=99 \text { chan } / \mathrm{km}
$$

4) the network bandwidth $Y_{p(M)}$ :

$$
Y_{p(M)}=Y(1-p)=9531.2 \cdot(1-0.005)=9483.56 \mathrm{Erl} ;
$$

5) the costs of arcs of each NA: $C_{3}=C_{4}=3044.5$, and the network cost as the sum of costs of the arcs of two NA:

$$
C=C_{3}+C_{4}=3044.5+3044.5=6089
$$

6) the parameters of reliability:

- between DSS in their own NA:

$$
\begin{gathered}
\pi=1-q^{3}=1-0.1^{3}=0.999 \\
\rho_{\max }=1-q=1-0.1=0.9
\end{gathered}
$$

- between a DSS of its own NA and an IMN of another NA:

$$
\begin{aligned}
& \pi=1-q=1-0.1=0.9 \\
& \rho=1-q=1-0.1=0.9
\end{aligned}
$$

- between DSS of different NA:

$$
\begin{gathered}
\rho_{\min }=(1-q) \cdot(1-q)=(1-0.1) \cdot(1-0.1)=0.81 ; \\
\pi_{\min }=1-q=1-0.1=0.9 .
\end{gathered}
$$

Similarly, we have performed the calculation of parameters for the local telephone network with the incoming (IMN) and outgoing (OMN) message nodes (Fig.3e). The network contains two nodal zones (NZ) of four DSS in each that are connected by the principle "each with each", and the DSS of different NZs are connected through the OMN of their zones and the IMN of the other zone. The calculation results are given in Table 9. Currently, the most widespread structure of the PSTN is a circular one represented in Fig. 3h. For this network, we have calculated [6]:

1) the total arcs lengths: $L=26 \mathrm{~km}$;
2) the calculation results showed that the number of channels in the arcs differs slightly, and for that reason, the maximum quantity of channels in the arc ( $V_{l m}=1560$ channels) was chosen to determine the number of channels in the network. In this case, the number of channels in the network equals:

$$
V=N_{b} \cdot V_{l m}=8 \cdot 1560=12480 \text { channels; }
$$

where $N_{b}$ is the arcs number in the network;
3) the network bandwidth in chan $/ \mathrm{km}$ :

$$
E=\frac{V}{L}=\frac{12480}{26}=480 \mathrm{chan} / \mathrm{km} ;
$$



Fig. 3. Public Switched Telephone Network structures.


Fig. 3. Public Switched Telephone Network structures.


Fig. 3. Public Switched Telephone Network structures.
4) the network bandwidth $Y_{p(M)}$ :

$$
\begin{gathered}
Y_{p(M)}=Y(1-p)=11913.016 \cdot(1-0.005)= \\
=11853.45 \mathrm{Erl}
\end{gathered}
$$

5) the network costs, which equals the sum of costs all the arcs: $C=33884.57$;
6) the parameters of reliability:

$$
\begin{gathered}
\rho_{\max }=1-q=1-0.1=0.9 ; \\
\rho_{\min }=(1-q)^{7}=(1-0.1)^{7}=0.478 ; \\
\pi=1-q^{2}=1-0.1^{2}=0.99 .
\end{gathered}
$$

The network of the "circle" type has two ways between any two nodes, therefore if the long way (7 arcs) fails, information will be transmitted through another way ( 1 arc ) with $\rho_{\max }=0.9$. If two ways of transmission in this network are taken into account, one way will have 4 arcs, and the other -3 arcs. Then for the network of the "circle" type:

$$
\rho_{\min }=(1-q)^{4}=(1-0.1)^{4}=0.656 .
$$

For a comparative analysis of the efficiency of telephone networks with different structures, all the obtained parameters of the investigated networks are tabulated in Table 9.

Table 9
The generalized efficiency parameters of telephone networks with different structures

| Cully-connected | Star | Linear | Radial-nodal | LTN with <br> IMN | LTN with <br> IMN and <br> OMN | Circle |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L, \mathrm{~km}$ | 309.6 | 30 | 27 | 22 | 108.2 | 103.8 | 26 |
| $V$, chan. | 7520 | 6624 | 11780 | 9192 | 10680 | 14066 | 12480 |
| $E, \mathrm{chan} / \mathrm{km}$ | 25 | 221 | 437 | 417 | 99 | 137 | 480 |
| $Y_{p(M)}, \operatorname{Erl}$ | 6227.52 | 6227.52 | 11288.6 | 8748.47 | 9483.56 | 12739.6 | 11853.45 |
| C | 7813 | 2628 | 3380 | 2338 | 6089 | 6275 | 3480 |
| $\rho_{\max }$ | 0.9 | 0.81 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| $\rho_{\min }$ | 0.478 | 0.81 | 0.43 | 0.656 | 0.81 | 0.729 | 0.656 |
| $\pi_{\max }$ | 0.9999999 | 0.9 | 0.9 | 0.9 | 0.999 | 0.999 | 0.99 |
| $\pi_{\min }$ | 0.9999999 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.99 |

## 5. Conclusion

To do the research into the efficiency of telephone networks, seven most widespread structures are built in the paper. The sizes of territory covered with telephone networks with different structures are calculated. The geometrical method intended for telephone network
modelling is justified. It was used to build a territory divided into eight elements. Using the arithmetic method, the given eight DSS were placed on the territory built. It was determined that

- the fully-connected network has a maximum coefficient of communication reliability and the longest
linelenghts, which are by a factor of ten longer than the lengths of tree-like and "circle" networks. It also has a less bandwidth capacity of one kilometer of the network (a 25 chan. $/ \mathrm{km}$ vs 480 chan./ km in a "circle" network), and high cost ( 7813 vs 3480 in a "circle" type). Such networks are reasonable in providing high communication reliability on small territories when there is a considerable network traffic and not more than five - six nodes.
- the tree-like network structures as compared with the fully-connected ones have by a factor of ten shorter linelengths; a 20-25 times greater bandwidth of one kilometer of the network; 2-2.5 times less cost, and far less reliability of communication. The advantage of the tree-like networks is their variety of structures, from which it is possible to choose the ones with necessary parameters.
- the combined structures by their parameters are intermediate between the fully-connected and tree-like structures: their linelength is 3 times less than in the fully-connected and 3.82 times greater than in the tree-like networks. The bandwidth in the channels of one-kilometer of the network is 5.5 times greater than in the fully-connected and 3.2 times less than in the tree-like networks. The cost of such a network is higher than the fully-connected one only by $25 \%$, but $2-2.8$ times higher than that of the tree-like. The reliability of the combined structures is lower than that of the fully-connected ( 0.729 vs 0.9 ) and star-like ( 0.729 vs 0.81 ) networks, but higher than of the linear and radial-nodal networks ( 0.729 vs 0.43 and 0.656 respectively). The reliability of connections between the nodes of the zone in the combined structures is higher than in the tree-like structure ( 0.999 vs 0.9 ), and between the nodes of different zones is as high as in the tree-like structure (0.9). It is reasonable that the combined networks be used on large territories, with a heavy network traffic and average requirements to reliability.
- the peculiarity of the "circle" type networks is the two ways between any two nodes, that influences their parameters. In this network, the length of the lines, total number of channels, bandwidth of onekilometer distance, general bandwidth, cost of the network, and reliability of the ways are the same as in the tree-like networks, but the probability of a connection failure is 10 times less than in the tree-like networks ( 0.01 vs 0.1 ). By the reliability of connection, the networks of "circle" type are inferior only to the fully-connected networks. Such networks are universal and have wide applications in the municipal telephone networks of large capacity. The
"circle" type structures have limited applications in the networks that cover large areas and have a heavy traffic leading to an increase in the transit arcs load that in turn generates demands on the capacity of the channel-forming equipment. Most rationally, these structures are used for the digitization of active analogue telephone networks.

The results we obtained in the course of reseach into efficiency, give the opportunity when planning and desinging telephone networks to choose such network structures, which are the best to meet the requirements established, and for the existing structure - to define its efficiency.

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## ДОСЛІДЖЕННЯ ЕФЕКТИВНОСТІ ТЕЛЕФОННИХ МЕРЕЖ ІЗ РІЗНИМИ СТРУКТУРАМИ

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З розвитком мереж зв’язку розвивалися й удосконалювалися структури мереж. Сьогодні перспективний розвиток телефонних мереж усіх рівнів їєрархії передбачає дослідження структур для їх побудови.

Метою цієї роботи є побудова й аналіз ефективності найбпоширеніших структур телефонних мереж загального користування згідно 3 визначеними критеріями ефективності. У роботі поставлені завдання: 1) дослідити найпоширеніші структури теле-

фонних мереж та побудувати їх на конкретній території з урахуванням вихідних даних і методів розташування вузлів; 2) обгрунтувати параметри ефективності телефонних мереж; 3) розрахувати вибрані параметри для побудованих структур i провести кількісний та якісний аналіз отриманих результатів. Актуальність цієї роботи пояснюється як відсутністю в літературі кількісного дослідження параметрів ефективності телефонних мереж з різними структурами, так і потребами практики, пов'язаними з широким застосуванням різних структур мереж зв'язку.


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