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DEVELOPMENT OF A METHODICS FOR IMPROVING THE ACCURACY OF DETERMINATION OF SPATIAL COORDINATES OF OBJECT POINTS DURING AIR SURVEILLANCE FROM A UAV

Purpose. The purpose of the paper is to develop an optimal algorithm that will increase the accuracy of determining the coordinates of the terrain when applying the aerial process using an unmanned aerial vehicle (UAV). **Method.** The study performs the minimization of function based on the condition of collinearity. It clarifies the elements of external orientation (EZO) of digital images and leads to an increase in the accuracy of the spatial coordinates of the points of objects. The proposed function is the sum of the squares of the differences between the calculated and measured reference points on the corresponding digital images. The sequence of implementation of the proposed algorithm includes taking into account the condition of the minimum of this function, which allows obtaining a system of six nonlinear equations for EZO. The process of determining EZO is performed in two ways: in the first case, the function G is minimized directly by one of the numerical methods, and in the second – obtained as a solution of a system of equations, which gives refined EZO values based on initial approximations obtained directly from UAV telemetry. Modified conditions of the minimum of the function G, in which there are no differentiation operations, are used to control the accuracy of EZO determination. As a result, we obtain the final values of the EZO at the time of shooting. **Results**. An algorithm has been developed and tested on mock-ups on real examples, which allows to increase the accuracy of calculating the coordinates of terrain points when using UAVs for the aerial photography process. **Scientific novelty. The research obtained the f**ormulas, which increase the accuracy of creating topographic materials by digital stereophotogrammetric method. **Practical significance**. The implementation of the developed algorithm will significantly increase the accuracy of processing large-scale orthophotos and topographic plans created on the basis of aerial photography from UAVs.

Key words: unmanned aerial vehicle, elements of external orientation, partial derivatives, nonlinear equations.

Introduction

In choosing the parameters of digital shooting from UAVs, in contrast to manned aerial photography, it is necessary to take into account a large number of additional conditions, calibration of shooting systems, local weather conditions, local parameters of altitude and speed during aerial photography, flight time, aerodynamic characteristics of UAVs. Particular attention in foreign and domestic publications is paid to the accuracy of determining spatial coordinates, which is confirmed by a number of scientific studies.

Most often, the preliminary calculation of accuracy is performed according to the simplest scheme and simplified methods, considering the resolution of the digital camera to be the main factor [Mikhailov, 2012], which is absolutely unacceptable. A more detailed overview of all factors and calculation of their impact is given in [Bosak, 2012], but the author calculated the accuracy of determining spatial coordinates by very approximate expressions, which does not fully assess the results of research.

[Schultz, et al., 2015] presents a more rigorous method for performing a preliminary calculation of the accuracy of determining the coordinates of points on aerial photography materials using unmanned aerial vehicles (UAVs). The authors developed a mathematical model for determining the coordinates using a pair of aerial photographs, taking into account the use of GNSS data to determine the coordinates of the centers of photography and correction of the inertial navigation system (INS). The authors find the angular elements of the external orientation using INS, applying the transformation algorithm of the correlation matrices used to perform the previous calculation of accuracy. The expression method is used to calculate the effect of systematic errors. The obtained strict mathematical expressions made it possible to perform a study of a priori accuracy of

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determining coordinates for different conditions of aerial photography. An experiment performed calculation of the accuracy of aerial photography for UAVs with typical characteristics used to create topographic maps and plans. This approach, proposed by the authors, is interesting, but ineffective. It does not take into account the influence of errors of external orientation elements and changes in elevations between the centers of photography. This will not adequately assess the accuracy of determining the coordinates of points.

The publication [Bezmenov, & Safin, 2019] considers the approach to determine the accuracy of the photogrammetric serif for an arbitrary case of aerial photography. To this end, we obtain analytical formulas for calculating the errors of spatial coordinates for any values of the elements of internal and external orientation of the camera. The formulas allow us to calculate the contribution of the error of any of the model parameters to the accuracy of spatial coordinates. In another work [Bezmenov, & Safin, 2019], the authors improved this approach, which provides a solution to the problem of accuracy of photogrammetric serif using the system of Euler angles. Researches allow us to note sufficient universality of the general approach to the specified task. However, the results showed that the use of the classical approach to the problems of estimating the accuracy of determining spatial coordinates by the method of photogrammetric serif leads to unjustifiably optimistic evaluation of numerous experiments.

The article [Yanchuk, & Trokhymets, 2017] covers estimation of the possibility of using unmanned aerial photography materials obtained from non-professional UAVs in order to create a topographic and geodetic basis for the development of master plans for settlements. Accuracy assessment was investigated by comparing the coordinates of control points determined from the constructed orthophoto with the coordinates obtained from the results of GNSS observations in the field. Based on the deviations of the coordinates of the control points, the mean square errors of the plan and altitude position of the points (0.10 m, 0.12 m, 0.18 m) were calculated. Their values meet the requirements of the instructions for compiling topographic and cadastral plans at a scale of 1: 2000. It should be noted that the article does not analyze the factors that may affect the quality of the obtained data. It also does not make an a priori assessment of the accuracy of determining the coordinates of the points of objects.

he article [Simeenev, & Tarasova, 2012] considers the reliability of the solution of the inverted photogrammetric serif when using different initial dependencies between coordinates of points of the district and their image on a picture. The paper presents the data characterizing the reliability of the decision of a problem on analytical models of plain terrain and a picture of scale 1: 10000. Based on the analysis of singular numbers, a method for determining the optimal scaling factor is proposed. According to the authors, it ensures the matrix of a system of normal equations and provides the reliability of the decision of the problem. It is shown that the analysis of the minimum values of the conditionality numbers allows us to choose the optimal type of initial dependences.

The linear solution of direct and inverse photogrammetric serifs is described in [Tsvetkov, 2011]. The coordinates of reference points of the terrain and the coordinates of their images in the photo are set as the source data. The author emphasizes that his proposed method does not impose restrictions on the angle of inclination of the photo. It also does not require preliminary values of the elements of internal orientation. In general, it allows solution of the collinear serif, that is straight serif, obtained by the coordinates of an unlimited number of images with the same and with different elements of internal orientation. Studies have shown that such a serif is solved in violation of the condition of coplanarity, which is mandatory in the classical approach. Another work [Korshunov, et al., 2013] investigates the method of non-central inverted photogrammetric serif and variants of incorrect solutions of this problem. An interesting solution to this problem is highlighted in the publication [Kim Hon Ir, et al., 2017], which presents an algorithm for solving the collinearity equation based on the quaternion algorithm. Determining the elements of external orientation regardless of the individual quaternions, the angle of inclination and the initial value of the unknown parameter in accordance with the analytical processing of aerial photographs taken by unmanned aerial vehicles, is based on the reliability of the proposed method by experimental calculations. The same idea was supported by another author

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[Bezmenov, & Safin, 2014]. He considered the equation of collinearity on the basis of the apparatus of quaternion algebra, where he replaced the angular elements of external orientation (Euler angles) with a quaternion.

In the publication [Silva, A. & Silva, D. 2015], the collinearity equation is used. It gives the rotations according to the Cartesian axis with the Euler angles. It is indicated that there may be angle combinations that leave the rotation matrix unstable and thus, the solution may not converge or even be undefined. This problem has been solved with the substitution of Euler angles by quaternions. Programs with iterative and direct methods with the substitution of Euler angles by quaternaries were implemented in order to compare with the colinearity method.

The work [Mazaheri, & Habib, 2015] introduces three quaternion-based approaches to solve the single photo resection (SPR) problem. The first two are based on projective transformation and DLT coefficients, the third general approach is based on quaternion. The approach solves the rotation matrix by enforcing a simple geometric constraint. It does not need partial derivative calculation or any complex computation.

Possibilities of using UAVs for modern combat operations, reconnaissance of the area are covered in the article [Chernyshev, & Kucenko, 2018]. In order to improve the determination of the coordinates of unmanned aerial vehicles in the area of the anti-terrorist operation, the authors assessed the accuracy of determining the location of radio emission targets by difference-range method in a movable passive radar system based on short-range anti-aircraft systems. The results of the study showed that the errors of measurement of coordinates in the difference-range method are insignificant and at some positions of the UAV are significant compared to its size. The obtained dependencies allow us to choose the optimal location of combat vehicles from passive direction finders in terms of obtaining the minimum errors of the UAV coordinates. Determination of spatial coordinates in the work was performed taking into account the estimation of the root mean square error in the proposed coordinate system by the method of statistical tests with a sample size of N = 500. It provided the necessary accuracy of the results to solve military problems. However, this accuracy does not meet the requirements of topographic shooting.

The article [Berezina, et al., 2018] analyzed the coordinate reference of images obtained with the help of UAVs. The authors developed a new method of snapping images by elements of external orientation to determine the spatial coordinates of the object. They were calculated using reference points at the allowable value of the deviation error from the actual position of the reference points no more than 0.5 m. The method considered in the work uses the simplex method of the deforming polygon. It allowed us to restore the true values of the elements of the external orientation, which increased the accuracy of determining the geographical coordinates of the points in the image.

The paper [Babinec, & Apeltauer, 2016] focuses on geometrical properties of error propagation of the object position estimation based on homography estimation between the projection plane. We decided to use uncertainty propagation analysis based on the Monte Carlo method, due to non-linearity caused by the use of position estimation from aerial imagery captured by a low-cost action camera mounted on low-flying UAVs.

Purpose

The purpose of the paper is to develop an optimal algorithm that will increase the accuracy of determining the coordinates of the area when applying the aerial process using a UAV.

Method

The technique is to minimize the function built on the condition of collinearity. It clarifies the elements of external orientation (EZO) of digital images. This in turn leads to increased accuracy of the spatial coordinates of the points of objects. Moreover, the proposed function is the sum of the squares of the differences between the calculated

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and the observation data of the reference points on the corresponding digital images. The sequence of implementation of the proposed algorithm consists of taking into account the condition of the minimum of this function, which allows us to obtain a system of six nonlinear equations for EZO. The process of determining EZO is performed in two ways: in the first case, the function G is minimized directly by one of the numerical methods; and in the second case, it is obtained as a solution of the equations system, which gives refined EZO values based on initial approximations obtained directly from UAV telemetry. Modified conditions of the minimum of the function G that lack differentiation operations are used to control the accuracy of EZO determination. As a result, we obtain the final values of the EZO at the time of the shooting.

Results

The paper proves the accuracy of the proposed method, considering the essence of the classical method. It is known that if it is not possible to determine the EZO α, ω, κ, X_S, Y_S, Z_S, by the hardware method, then the only way to obtain them is the method based on the condition of collinearity. Its essence is to select such parameters for which the difference between the deviations of the calculated and measured coordinates in digital images is minimal. It is known [Dorozhynsky, & Tukay, 2008] that the coordinates in the images are determined by the coordinates in the field X_i, Y_i, Z_i according to the following formulas:

$$x_i - x_0 = -f\,\frac{a_1(X_i - X_S) + b_1(Y_i - Y_S) + c_1(Z_i - Z_S)}{a_3(X_i - X_S) + b_3(Y_i - Y_S) + c_3(Z_i - Z_S)} \quad (1)$$

$$y_i - y_0 = -f\,\frac{a_2(X_i - X_S) + b_2(Y_i - Y_S) + c_2(Z_i - Z_S)}{a_3(X_i - X_S) + b_3(Y_i - Y_S) + c_3(Z_i - Z_S)}$$

To simplify the presentation of the material we will take

$$x_i - x_o \sim x_i,\ y - y_o \sim y_i,$$

$$u_i \sim X_i - X_S,\ v_i \sim Y_i - Y_S,\ w_i \sim Z_i - Z_S. \quad (2)$$

Then we get:

$$\tilde{x}_1 = -f\,\frac{a_1u_i + b_1v_i + c_1w_i}{a_3u_i + b_3v_i + c_3w_i}, \tilde{y}_1 = -f\,\frac{a_2u_i + b_2v_i + c_2w_i}{a_3u_i + b_3v_i + c_3w_i},$$

$$\tilde{z}_1 = a_3u_i + b_3v_i + c_3w_i. \quad (3)$$

The choice of EZO elements α, ω, κ, X_S, Y_S, Z_S is determined by the condition

$$G(\alpha,\omega,\kappa,X_S,Y_S,Z_S) = \sum_{i=1}^{n}(x_i - \tilde{x}_1)^2 + (y_i - \tilde{y}_1)^2 \to \min \quad (4)$$

and the estimation of accuracy is calculated by the formula

$$m_G = \sqrt{\sum_{i=1}^{n}(x_i - \tilde{x}_1)^2 + (y_i - \tilde{y}_1)^2}. \quad (5)$$

The traditional approach is to decompose the right-hand sides of expressions (1) into the Taylor series by EZO and to choose values at which equations (1) hold or are close to zero. The minimum number of points *n* required to determine the corrections is determined by number three. In practice, their number is taken larger, that is n> 3. Therefore, the system of equations that arises during equilibration is solved using the least-squares method. In this approach, an important factor is the choice of the decomposed point in the row, which is interpreted as the initial approximation. The calculation algorithm is as follows: for a certain initial approximation, a system of linear equations is determined, from which the EZO corrections are calculated. Next, the refined values are taken as the initial and the calculations are re-preformed, which end when the previous and subsequent values coincide within the required number of characters.

In our opinion, the accuracy of the EZO definition is lost during the series decomposition, which in turn leads to a decrease in the accuracy of determining the coordinates of the points of the terrain objects. This requires other approaches. In particular, the following studies have identified the possibility of improving the accuracy of EZO.

One option for another approach may be to rethink the search for extreme values. The content of the problem is to find the minimum of the function *G*. A necessary condition for its existence [Shkil, 2015] is the equality with zero partial derivatives of the function *G*:

$$\begin{cases} \dfrac{\partial G}{\partial \alpha} = 0 & \dfrac{\partial G}{\partial X_S} = 0 \\ \dfrac{\partial G}{\partial \omega} = 0 & \dfrac{\partial G}{\partial Y_S} = 0 \\ \dfrac{\partial G}{\partial \kappa} = 0 & \dfrac{\partial G}{\partial Z_S} = 0 \end{cases} \quad (6)$$

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We present the function G in extended form:

$$G(\alpha,\omega,\kappa,X_S,Y_S,Z_S)=\sum_{i-1}^{n}\left(\left(x_i+f\,\frac{a_1(X_i-X_S)+b_1(Y_i-Y_S)+c_1(Z_i-Z_S)}{a_3(X_i-X_S)+b_3(Y_i-Y_S)+c_3(Z_i-Z_S)}\right)^2+\left(y_i+f\,\frac{a_2(X_i-X_S)+b_2(Y_i-Y_S)+c_2(Z_i-Z_S)}{a_3(X_i-X_S)+b_3(Y_i-Y_S)+c_3(Z_i-Z_S)}\right)^2\right) \tag{7}$$

Differentiation of expression (7) by variables X_S, Y_S, Z_S gives the last three systems (6):

$$\begin{aligned}
\frac{\partial G}{\partial X_S}&=-\sum_{i=1}^{n}(x_i-\tilde{x}_1)\frac{fa_1}{\tilde{z}_1}-(x_i-\tilde{x}_1)\frac{\tilde{x}_1a_3}{\tilde{z}_1}+(y_i+\tilde{y}_1)\frac{fa_2}{\tilde{z}_1}-(y_i+\tilde{y}_1)\frac{\tilde{y}_1a_3}{\tilde{z}_1}\\
\frac{\partial G}{\partial Y_S}&=-\sum_{i=1}^{n}(x_i-\tilde{x}_1)\frac{fb_1}{\tilde{z}_1}-(x_i-\tilde{x}_1)\frac{\tilde{x}_1b_3}{\tilde{z}_1}+(y_i+\tilde{y}_1)\frac{fb_2}{\tilde{z}_1}-(y_i+\tilde{y}_1)\frac{\tilde{y}_1b_3}{\tilde{z}_1}\\
\frac{\partial G}{\partial Z_S}&=-\sum_{i=1}^{n}(x_i-\tilde{x}_1)\frac{fc_1}{\tilde{z}_1}-(x_i-\tilde{x}_1)\frac{\tilde{x}_1c_3}{\tilde{z}_1}+(y_i+\tilde{y}_1)\frac{fc_2}{\tilde{z}_1}-(y_i+\tilde{y}_1)\frac{\tilde{y}_1c_3}{\tilde{z}_1}
\end{aligned} \tag{8}$$

The first three equations of system (6) are established by differentiation by additional variables-components of vectors $\vec{a}=(a_1,a_2,a_3)$, $\vec{b}=(b_1,b_2,b_3)$, $\vec{c}=(c_1,c_2,c_3)$, $\vec{a}=(a_1,a_2,a_3)$, $\vec{b}=(b_1,b_2,b_3)$, $\vec{c}=(c_1,c_2,c_3)$, $\vec{X}=(X_S,Y_S,Z_S)$, which are connected by a number of conditions

I $a_1^2+a_2^2+a_3^2=1$,

II $b_1^2+b_2^2+b_3^2=1$,

III $c_1^2+c_2^2+c_3^2=1$, $\lambda=(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6),X^{\leftrightarrow}$,

IV $a_1b_1+a_2b_2+a_3b_3=0$,

V $a_1c_1+a_2c_2+a_3c_3=0$,

VI $b_1c_1+b_2c_2+b_3c_3=0$.

In this statement, the problem is formulated as finding the minimum of the Lagrange function:

$$F(\vec{a},\vec{b},\vec{c},\vec{X},l_1,...,l_6)=\sum_{i=1}^{n}(x_i-\tilde{x}_1)^2+(y_i-\tilde{y}_1)^2+l_1(a_1^2+a_2^2+a_3^2-1)+l_2(b_1^2+b_2^2+b_3^2-1)+$$
$$l_3(c_1^2+c_2^2+c_3^2-1)+l_4(a_1b_1+a_2b_2+a_3b_3)+l_5(a_1c_1+a_2c_2+a_3c_3)+l_6(b_1c_1+b_2c_2+b_3c_3),$$

$$\frac{\partial F}{\partial a_1}=2\sum_{i=1}^{n}(x_i-\tilde{x}_1)\frac{u_if}{\tilde{z}_1}+2\lambda_1a_1+\lambda_4b_1+\lambda_5c_1=0,$$

$$\frac{\partial F}{\partial a_2}=2\sum_{i=1}^{n}(y_i-\tilde{y}_1)\frac{u_if}{\tilde{z}_1}+2\lambda_1a_2+\lambda_4b_2+\lambda_5c_2=0,$$

$$\frac{\partial F}{\partial a_3}=-2\sum_{i=1}^{n}((x_i-\tilde{x}_1)\frac{u_i\tilde{z}_1}{\tilde{z}_1}+(y_i-\tilde{y}_1)\frac{u_i\tilde{z}_1}{\tilde{z}_1})+2\lambda_1a_3+\lambda_4b_3+\lambda_5c_3=0.$$

Transforming the last three equations, we obtain:

$$\frac{\partial F}{\partial a_1}a_1+\frac{\partial F}{\partial a_2}a_2+\frac{\partial F}{\partial a_3}a_3=2\sum_{i=1}^{n}\frac{u_i}{\tilde{z}_1}((x_i-\tilde{x}_1)\,fa_1+(y_i-\tilde{y}_1)\,fa_2-a_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))+2\lambda_1=0,$$

$$\frac{\partial F}{\partial a_1}b_1+\frac{\partial F}{\partial a_2}b_2+\frac{\partial F}{\partial a_3}b_3=2\sum_{i=1}^{n}\frac{u_i}{\tilde{z}_1}((x_i-\tilde{x}_1)\,fb_1+(y_i-\tilde{y}_1)\,fb_2-b_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))+\lambda_4=0,$$

$$\frac{\partial F}{\partial a_1}c_1+\frac{\partial F}{\partial a_2}c_2+\frac{\partial F}{\partial a_3}c_3=2\sum_{i=1}^{n}\frac{u_i}{\tilde{z}_1}((x_i-\tilde{x}_1)\,fc_1+(y_i-\tilde{y}_1)\,fc_2-c_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))+2\lambda_5=0.$$

Similarly, differentiating by variables b_1,b_2,b_3, we find:

$$\frac{\partial F}{\partial b_1}b_1+\frac{\partial F}{\partial b_2}b_2+\frac{\partial F}{\partial b_3}b_3=2\sum_{i=1}^{n}\frac{v_i}{\tilde{z}_1}((x_i-\tilde{x}_1)\,fb_1+(y_i-\tilde{y}_1)\,fb_2-b_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))+\lambda_2=0,$$

$$\frac{\partial F}{\partial b_1}a_1+\frac{\partial F}{\partial b_2}a_2+\frac{\partial F}{\partial b_3}a_3=2\sum_{i=1}^{n}\frac{v_i}{\tilde{z}_1}((x_i-\tilde{x}_1)\,fa_1+(y_i-\tilde{y}_1)\,fa_2-a_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))+\lambda_4=0,$$

$$\frac{\partial F}{\partial b_1}c_1+\frac{\partial F}{\partial b_2}c_2+\frac{\partial F}{\partial b_3}c_3=2\sum_{i=1}^{n}\frac{v_i}{\tilde{z}_1}((x_i-\tilde{x}_1)\,fc_1+(y_i-\tilde{y}_1)\,fc_2-c_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))+\lambda_6=0.$$

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By variables c_1, c_2, c_3 we have:

$$\frac{\partial F}{\partial c_1}c_1 + \frac{\partial F}{\partial c_2}c_2 + \frac{\partial F}{\partial c_3}c_3 = 2\sum_{i=1}^{n}\frac{w_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fc_1 + (y_i - \tilde{y}_1)fc_2 - c_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) + \lambda_3 = 0,$$

$$\frac{\partial F}{\partial c_1}a_1 + \frac{\partial F}{\partial c_2}a_2 + \frac{\partial F}{\partial c_3}a_3 = 2\sum_{i=1}^{n}\frac{w_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fa_1 + (y_i - \tilde{y}_1)fa_2 - a_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) + \lambda_5 = 0,$$

$$\frac{\partial F}{\partial c_1}b_1 + \frac{\partial F}{\partial c_2}b_2 + \frac{\partial F}{\partial c_3}b_3 = 2\sum_{i=1}^{n}\frac{w_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fb_1 + (y_i - \tilde{y}_1)fb_2 - b_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) + \lambda_6 = 0.$$

We exclude variables $\lambda_4, \lambda_5, \lambda_6$ from the whole set of equations:

$$\begin{gathered}
\sum_{i=1}^{n}\frac{u_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fb_1 + (y_i - \tilde{y}_1)fb_2 - b_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) = \\
= \sum_{i=1}^{n}\frac{v_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fa_1 + (y_i - \tilde{y}_1)fa_2 - a_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)), \\
\sum_{i=1}^{n}\frac{u_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fc_1 + (y_i - \tilde{y}_1)fc_2 - c_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) = \\
= \sum_{i=1}^{n}\frac{w_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fa_1 + (y_i - \tilde{y}_1)fa_2 - a_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)), \\
\sum_{i=1}^{n}\frac{w_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fb_1 + (y_i - \tilde{y}_1)fb_2 - b_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) = \\
= \sum_{i=1}^{n}\frac{v_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fc_1 + (y_i - \tilde{y}_1)fc_2 - c_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)).
\end{gathered} \tag{9}$$

Differentiation of the function F by variables λ_i and formula (9), and after their simplifications, leads to a system of nine nonlinear equations with nine unknowns:

$$\begin{cases}
\sum_{i=1}^{n}\frac{u_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fb_1 + (y_i - \tilde{y}_1)fb_2 - b_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) = \\
= \sum_{i=1}^{n}\frac{v_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fa_1 + (y_i - \tilde{y}_1)fa_2 - a_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) \\
\sum_{i=1}^{n}\frac{u_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fc_1 + (y_i - \tilde{y}_1)fc_2 - c_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) = \\
= \sum_{i=1}^{n}\frac{w_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fa_1 + (y_i - \tilde{y}_1)fa_2 - a_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) \\
\sum_{i=1}^{n}\frac{w_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fb_1 + (y_i - \tilde{y}_1)fb_2 - b_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) = \\
= \sum_{i=1}^{n}\frac{v_i}{\tilde{z}_1}((x_i - \tilde{x}_1)fc_1 + (y_i - \tilde{y}_1)fc_2 - c_3((x_i - \tilde{x}_1)\tilde{x}_1 + (y_i - \tilde{y}_1)\tilde{y}_1)) \\
a_1^2 + a_2^2 + a_3^2 = 1 \\
a_1^2 + a_2^2 + a_3^2 = 1 \\
b_1^2 + b_2^2 + b_3^2 = 1 \\
a_1c_1 + a_2c_2 + a_3c_3 = 0 \\
b_1c_1 + b_2c_2 + b_3c_3 = 0 \\
c_1^2 + c_2^2 + c_3^2 = 1
\end{cases}$$

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In the given system it is possible to remove partially unknowns therefore we will come to three nonlinear equations. Supplemented by expressions (8) they give a system with unknowns $\alpha,\omega,\kappa,X_S,Y_S,Z_S$.

$$\begin{cases}
\sum_{i=1}^{n}(x_i-\tilde{x}_1)\frac{fa_1}{\tilde{z}_1}-(x_i-\tilde{x}_1)\frac{\tilde{x}_1a_3}{\tilde{z}_1}+(y_i+\tilde{y}_1)\frac{fa_2}{\tilde{z}_1}-(y_i+\tilde{y}_1)\frac{\tilde{y}_1a_3}{\tilde{z}_1}=0\\
\sum_{i=1}^{n}(x_i-\tilde{x}_1)\frac{fb_1}{\tilde{z}_1}-(x_i-\tilde{x}_1)\frac{\tilde{x}_1b_3}{\tilde{z}_1}+(y_i+\tilde{y}_1)\frac{fb_2}{\tilde{z}_1}-(y_i+\tilde{y}_1)\frac{\tilde{y}_1b_3}{\tilde{z}_1}=0\\
\sum_{i=1}^{n}(x_i-\tilde{x}_1)\frac{fc_1}{\tilde{z}_1}-(x_i-\tilde{x}_1)\frac{\tilde{x}_1c_3}{\tilde{z}_1}+(y_i+\tilde{y}_1)\frac{fc_2}{\tilde{z}_1}-(y_i+\tilde{y}_1)\frac{\tilde{y}_1c_3}{\tilde{z}_1}=0\\
\sum_{i=1}^{n}\frac{u_i}{\tilde{z}_1}((x_i-\tilde{x}_1)fb_1+(y_i-\tilde{y}_1)fb_2-b_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))=\\
\sum_{i=1}^{n}\frac{v_i}{\tilde{z}_1}((x_i-\tilde{x}_1)fa_1+(y_i-\tilde{y}_1)fa_2-a_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))\\
\sum_{i=1}^{n}\frac{u_i}{\tilde{z}_1}((x_i-\tilde{x}_1)fc_1+(y_i-\tilde{y}_1)fc_2-c_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))=\\
=\sum_{i=1}^{n}\frac{w_i}{\tilde{z}_1}((x_i-\tilde{x}_1)fa_1+(y_i-\tilde{y}_1)fa_2-a_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))\\
\sum_{i=1}^{n}\frac{w_i}{\tilde{z}_1}((x_i-\tilde{x}_1)fb_1+(y_i-\tilde{y}_1)fb_2-b_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))=\\
=\sum_{i=1}^{n}\frac{v_i}{\tilde{z}_1}((x_i-\tilde{x}_1)fc_1+(y_i-\tilde{y}_1)fc_2-c_3((x_i-\tilde{x}_1)\tilde{x}_1+(y_i-\tilde{y}_1)\tilde{y}_1))
\end{cases} \quad (10)$$

We use the system of equations (10) as a criterion for achieving the minimum of the function G. The advantage of this approach is the possibility of verification without finding derivatives. To confirm the above calculations, we test the proposed algorithm on the relevant examples. Table 1 shows the coordinates of reference points in the field measured in images obtained from UAVs. Table 2 presents EZO, calculated sequentially: I line – initial values, II line – adjusted values taking into account direct minimization, III line – specified as a solution of the system (6). The error value G decreases by an order of magnitude (eighth column, rows No. 2, 3).

Table 1

Coordinates of reference points in the field and their photogrammetric coordinates in the image

No.	$X\,(m)$	$Y\,(m)$	$Z\,(m)$	$x_л$ (mm)	$y_л$ (mm)
1	5455868.56	670717.53	574.45	10.666	14.473
2	5455710.54	670755.53	581.13	10.548	-11.845
3	5455757.67	670693.33	576.72	2.599	-1.760
4	5455834.278	670653.26	574.03	-0.642	12.248
5	5455702.22	670689.28	584.14	-0.296	-10.931
6	5455819.22	670627.87	573.77	-5.392	10.974
7	5455762.24	670673.22	575.95	-0.445	-0.182
8	5455771.55	670673.12	575.57	-0.,078	1.352
9	5455769.06	670691.81	576.07	2.818	0.142

Table 2

Elements of the external orientation of the left image

	α°	ω°	κ°	$X_S(m)$	$Y_S(m)$	$Z_S(m)$	$m_G\,(mm)$
I	5°22′40.5″	0°54′6.45″	14°44′20.18″	670650.10	5455759.04	784.62	9.260
II	5°22′40.,506″	0°54′4.23″	14°44′20.18″	670655.99	5455760.70	784.62	0.053
III	6°05′21.6″	1°21′49.10″	14°39′30.86″	670653.27	545575.89	785.00	0.006

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Checking condition (6) for our example shows significant deviations (Table 3) for the initial values (line I) and obtained with the minimization of the function (line II). The calculated values of EZO parameters from the system (6) give the fulfillment of this condition (line III). Therefore, the values of EZO are more real in solving our problem, although they are significantly different from the previous ones.

This is due to the following factors: firstly, the value of the function is minimal, and secondly, all partial derivatives are almost zero And this is a condition of a minimum of the function of many variables.

Table 3

The values of the derivatives of the function *G*

No.	$\frac{\partial G}{\partial \alpha}$	$\frac{\partial G}{\partial \omega}$	$\frac{\partial G}{\partial \kappa}$	$\frac{\partial G}{\partial X_S}$	$\frac{\partial G}{\partial Y_S}$	$\frac{\partial G}{\partial Z_S}$
I	-0.5176839	-0.1431993	0.0246195	-2.35232E-3	-6.54301E-4	-4.77721E-4
II	-4.58238E-3	-2.526614E-3	2.494619E-3	-8.25534E-6	-2.53483E-7	-4.61511E-5
III	-5.446717E-12	5.61220E-11	2.78960E-12	-4.27043E-14	2.39136E-13	1.05774E-13

As noted above, let us test the system (10) specifically on the example. The result for this example is given in the table 4 (columns 1–6 show the results of calculating the left parts of the system (10), and lines I–III are similar to Table 2, 3).

Table 4

The values of the left parts of the system (10) of the derivatives of the function G

No.	1	2	3	4	5
I	0.028	9.175E-3	0.153	-0.335	-9.059
II	-2.375E-3	-4.183E-4	0.028	-0.041	0.08
III	3.287E-12	-3.06E-12	-2.541E-11	2.385E-11	7.98E-11

Scientific novelty

The obtained formulas increase the accuracy of creating topographic materials by the digital stereophotogrammetric method.

Practical significance

The implementation of the developed algorithm will significantly increase the accuracy of processing large-scale orthophotos and topographic plans created on the basis of aerial photography from UAVs.

Conclusions

From the above studies we can draw the following conclusions:

1. The parameters of EZO are determined with the help of classical approach with insufficient accuracy required for the technological scheme of creating large-scale orthophotos.
2. Improving the accuracy of the determination of EZO is possible using the proposed algorithm, which is confirmed by the examples of the real material.
3. In further research, the authors plan to consider the possibility of using direct solutions of systems of nonlinear equations, which will help avoid additional conditions for finding EZO.
4. Authors in the future are planning to carry out thorough estimation of proposed methodology using sufficient control points.

REFERENCES

Babinec, A. & Apeltauer, J. (2016). On accuracy of position estimation from aerial imagery captured by low-flying UAVs. *International Journal of Transportation Science and Technology*, 5(3), 152–166. https://doi.org/10.1016/j.ijtst.2017.02.002. (in English).

Berezina, S., Logachov, S. & Solonets, O. (2018). Method of coordinate referencing of the images received from UAV, by elements of external orientation *Systemy ozbroyennya i viys'kova tekhnika*, 1(53), 76–83. doi: 10.30748/soivt.2018.53.11. (in Ukrainian).

Bezmenov, V. M. (2014). The use of quaternions in photogrammetry. *Proceedings of the Higher Educational Institutions. Izvestia vuzov. Geodesy and aerophotosurveying*, (5), 22–27. (in Russian)

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

Bezmenov, V. M., & Safin, K. I. (2019). Photogrammetric intersection. Accuracy estimation for an arbitrary case of aerial survey. *Proceedings of the Higher Educational Institutions. Izvestia vuzov. Geodesy and aerophotosurveying*, *63*(4), 400–406. doi: 10.30533/0536- 101X-2019-63-4-400-406. (in Russian)

Bezmenov, V. M. & Safin, K. I. (2019). The accuracy estimation of photogrammetric spatial intersection for random shooting case. The general approach to solving the problem. *Sovremennyye problemy distantsionnogo zondirovaniya Zemli iz kosmosa*, 16(6), 283–289. (in Russian).

Bosak K. (2012). Secrets of UAV photomapping by Krzysztof Bozak. 66p. URL: http://s3.amazonaws.com/DroneMapper_US/documentation/pteryx-mapping-secrets.pdf.

Chernyshev, M. & Kucenko, V. (2018). Assessment of accuracy of definition of position of the UAV by a difference-ranging method in the moving system of a passive radar-location in air defense complexes ground forces of small range. *Systemy ozbroyennya i viys'kova tekhnika*, 2(54), 61–66. doi:10.30748/soivt.2018.54.08. (in Ukrainian).

Dorozhyns′kyy, O. Tukay, R. (2008). Photogrammetry. −L′viv: Vydavnytstvo Natsional′noho universytetu “L′vivs′ka politekhnika”, 332 p.

Kim, H., Ryu, C., Kim, Z. & Zhen C. (2017). Study of the possibility of using quaternions to determine the parameters of exterior orientation in photogrammetry. *Nauchnyye issledovaniya*, 5(16), 85–89. (in Russian).

Korshunov, R. A., Noskov, V. V. & Pogorelov, V. V. (2013). Off-center photogrammetric inverse serif . *Izvestiya vysshikh uchebnykh zavedeniy. Geodeziya i aerofotos"yemka*, (5), 67–71. (in Russian).

Mazaheri, M. & Habib, A. (2015). Quaternion-Based Solutions for the Single Photo Resection Problem. *Photogrammetric Engineering & Remote Sensing*, 81(3), 209–217. (in English).

Mihaylov, A. P. (2012). Again about the choice of digital cameras to perform aerial survey with unmanned vehicles. In Proceedings of the 12th International Scientific Conference “From image to map: digital photogrammetric technologies”. (in Russian)

Schkil, M. I. (2005). Mathematical analysis u 2-kh tomakh Pidruchnyk u 2-kh ch., – 3-tye vydannya, pererobl. i dopovn., K.: Vyshcha shk., 447 s. (in Ukrainian).

Schultz, R. V., Voytenko, S. P., Krelshteinn, P. D. & Malina, I. A. (2015). The issue of calculating points positioning accuracy for aerial photographs from unmanned aerial vehicles. *Inzhenerna heodeziya*, (62), 124-136. (in Ukrainian).

Silva, A. M. & Silva, D. C. (2015). Resseção espacial em fotogrametria com quatérnios (The photogrammetric spatial resection using quaternion). *Boletim de Ciências Geodésicas*, 21(4), 2015, 750–764. (in Spanish).

Simineev, A. A. & Tarasova, E. I. (2012). Photogrammetric resection: reliability of task solution. *Vestnik Sankt-Peterburgskogo universiteta. Seriya 7: Geologiya. Geografiya*, (4), 129–134. (in Russian).

Tsvetkov, V. Ja. (2011). Linear photogrammetry an intersection. *Nauki o Zemle: mezhdunarodnyy nauchno-tekhnicheskiy i proizvodstvennyy zhurnal*, (2), 44–46. (in Russian).

Yanchuk, R. M. & Trokhymets, S. M. (2017). Creating cartographic basis for developing master plans of settlements on materials of aerial surveys using unspecialized inexpensive UAV. *Visnyk Natsional'noho universytetu vodnoho hospodarstva ta pryrodokorystuvannya. Seriya «Tekhnichni nauky»*, 1 (77), 32–39. (in Ukrainian)..

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РОЗРОБКА МЕТОДИКИ ПІДВИЩЕННЯ ТОЧНОСТІ ВИЗНАЧЕННЯ ПРОСТОРОВИХ КООРДИНАТ ТОЧОК ОБ'ЄКТІВ ПРИ АЕРОЗНІМАННІ З БПЛА

Мета. Розробити оптимальний алгоритм, завдяки якому вдасться підвищити точність визначення координат місцевості при застосуванні аерознімального процесу з допомогою безпілотного літального апарату (БПЛА). **Методика.** Виконується мінімізація функції побудованої на підставі умови колінеарності, що дає уточнення елементів зовнішнього орієнтування (ЕЗО) цифрових зображень, а це у свою чергу приводить до підвищення точності просторових координат точок об'єктів. Причому, запропонована функція – це сума квадратів різниць між вирахуваними та даними спостережень опорних точок на відповідних цифрових зображеннях. Послідовність реалізації запропонованого алгоритму полягає в тому, що урахування умови мінімуму цієї функції дає можливість отримати систему шести нелінійних рівнянь стосовно ЕЗО. Процес визначення ЕЗО виконується двома способами: в першому випадку функцію G мінімізуємо безпосередньо одним з чисельних методів, а в другому – одержуємо як розв'язок системи рівнянь, що дає уточнені значення ЕЗО на підставі початкових наближень, отриманих безпосередньо з телеметрії БПЛА. Для контролю точності визначення ЕЗО застосовуються видозмінені умови мінімуму функції G в яких відсутні операції диференціювання. В результаті, отримаємо остаточні значення ЕЗО в момент знімання. **Результати.** Розроблений і апробований на макетних на реальних прикладах алгоритм, який дозволяє підвищити точність обчислення координат точок місцевості при застосуванні БПЛА для аерознімального процесу. **Наукова новизна.** Отримані формули, за допомогою яких підвищується точність створення топографічних матеріалів цифровим стереофотограмметричним методом. **Практична значущість**. Впровадження розробленого алгоритму дасть змогу суттєво підвищити точність опрацювання великомасштабних ортофотопланів та топографічних планів створених за матеріалами аерознімання з БПЛА.

Ключові слова: безпілотний літальний апарат, елементи зовнішнього орієнтування, часткові похідні, нелінійні рівняння.

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