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APPLICATION OF THE FREQUENCY SYMBOLIC METHOD FOR ANALYSIS OF LINEAR PERIODICALLY-TIME-VARIABLE CIRCUITS IN THE TIME DOMAIN

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Abstract: This paper focuses on the problem of using the symbolic frequency method for the determining the time dependency of output signals of linear periodically-time-variable circuits by applying the inverse Fourier transform (for a steady-state mode) or the Laplace transform (for a transition mode) to the images of these signals obtained using the transfer functions.

The frequency symbolic method allows us to calculate the conjugate parametric transfer functions of linear periodically-time-variable circuits. Such transfer functions link the input signals with the output signals in the form of approximating polynomials of Fourier in trigonometric or complex form. By the same polynomials of Fourier are approximated the normal parametric transfer functions which are the basis of assessment of the asymptotic stability of the circuit. To determine the conjugate and normal parametric transfer functions, there are the appropriate functions in the system of software functions Multivariate Analysis and Optimization of the Parametric Circuits (MAOPC), The system of software functions MAOPCs based on the frequency symbolic method and is implemented in the environment of MATLAB software.

The paper presents the results of computational experiments obtained by the system of software functions MAOPCs which show the adequacy of the definition of steady-state and transient modes of linear periodically-time-variable circuits in the time domain using the transfer functions determined by frequency symbolic method.

The coincidence of the results obtained by the system of software functions MAOPCs and the Micro-Cap7.0 program demonstrates the adequacy of applying the inverse Fourier and Laplace transforms for the investigations of linear periodically-time-variable circuits in the steady and transient modes in the MAOPCs environment.

Key words: time analysis, conjugate parametric transfer function, frequency symbolic method, inverse Fourier transform, inverse Laplace transform, the system of software functions MAOPCs.

1. Introduction

In several works [1, 2, 3] t$\hat{W}(s,t)$he symbolic frequency method (FS-method) of the analysis of steady-state linear periodically-time-variable (LPTV) circuits has been developed and studied. It is based on the definition of their conjugate transfer functions $W(s,t)$ [3, 4] in the form of the approximated trigonometric Fourier polynomials

$$\hat{W}(s,t) = W_0(s) + \sum_{i=1}^{k} \begin{bmatrix} W_{ci}(s)\cos(i\Omega t) + \\ +W_{si}(s)\sin(i\Omega t) \end{bmatrix} \quad (1)$$

or as complex polynomials

$$\hat{W}(s,t) = W_{\pm 0}(s) + \sum_{i=1}^{k} \begin{bmatrix} W_{-i}(s)\cdot\exp(-ji\Omega t) + \\ +W_{+i}(s)\cdot\exp(+ji\Omega t) \end{bmatrix}, \quad (2)$$

where $W_0(s) = \dfrac{w_0(s)}{d(s)}, W_{ci}(s) = \dfrac{w_{ci}(s)}{d(s)}, W_{si}(s) = \dfrac{w_{si}(s)}{d(s)}$,

$W_{\pm 0}(s) = \dfrac{w_0(s)}{\Delta(s)}, W_{-i}(s) = \dfrac{w_{-i}(s)}{\Delta(s)}, W_{+i}(s) = \dfrac{w_{+i}(s)}{\Delta(s)}$ are the time-independent t fractional rational functions of a complex variable s, k is the number of harmonics in the approximation, $\Omega = 2p/T, T$ is the period of the parameter change of the parametric element of the circle under the action of the pump signal.

Let us note that the fractional rational functions, in addition to the symbolic variable s, can contain the parameters of the circuit elements given in the symbolic form. Multiple substitutions of different numerical values for these symbols, as a rule, are executed in multiple tasks at the stages of optimization, statistical analysis or synthesis of designed devices.

In [1, 2, 3, 5] the appropriateness of the FS-method for computing frequency characteristics is convincingly shown. In [1, 2, 6] it is also shown that the FS-method is adequate and provides a solid basis for the formation of sensitivity functions or the objective function when solving statistic tasks or carrying out the optimization of the LPTV circuits. An important condition for the adequacy of the FS-method is the choice of the sufficient magnitude of the value k in the approximating expressions (1), (2).

In our opinion, one of the unsolved tasks of using the FS-method remains the problem of determining the time dependency of output signals of the LPTV circuits by applying the inverse Fourier transform (for the steady-state mode) or the Laplace transform (for the transition mode) to images of these signals obtained with the use of the transfer functions. It should be noted that, as we know, such a task has not been posed and solved yet, due to the fact that no mathematical tools to determine such transfer functions and no appropriate software have been existed till now. However, with the advent of the FS-method and the system of software functions Multivariate Analysis and Optimization of the Parametric Circuits (MAOPC) based on it [6] such a possibility appeared, and it will be discussed below.

In connection with the above, the aim of this work is to study the adequacy of the calculation of the time dependencies of voltages and currents in the LPTV circuits in the presence of the frequency symbolic transfer functions defined by the FS-method. In our opinion, this way of calculating time dependences at the presence of multiple changes of the input signals and the parameters of the circuit elements can be more acceptable than, for example, their determination by the numerical methods, in particular, by the Micro-Cap program [7].

2. The main part

It is known [4], that by analogy with the linear circuits with constant parameters the output variable y(t) of the LPTV circuit in the time domain t can be represented in the form of the inverse Fourier transform

$$y(t) = \frac{1}{2 \cdot p} \cdot \int_{-\infty}^{\infty} W(jw,t) \cdot X(jw)\, e^{jw \cdot t}\, dw, \qquad (3)$$

or the Laplace transform

$$y(t) = \frac{1}{2 \cdot p} \cdot \int_{c-j\cdot\infty}^{c+j\cdot\infty} W(s,t) \cdot X(s)\, e^{s \cdot t}\, ds, \qquad (4)$$

where $X(jw)$ and $X(s)$ are the images of the input signal $x(t)$ applied to the circuit, obtained by the Fourier transform and the Laplace transform, respectively.

Let us note that:

a) expression (3) determines the time dependence of the output signal in the steady-state mode of a circuit, with the periodic input signal applied and within the time period from $t = -\infty$ to $t = +\infty$;

b) expression (4) defines the output signal in the mode of the transition process within the time period from $t = 0$ to $t = +\infty$;

c) in expressions (3) and (4) the inverse transformations of the images of the output signals are performed

$$Y(jw,t) = W(jw,t) \cdot X(jw), \qquad (5)$$

$$Y(s,t) = W(s,t) \cdot X(s), \qquad (6)$$

respectively.

Thus, by the determined images of the output signal in the form of (5) or (6), and by the formulas for its decomposition into simple fractions known in the operational calculus [8], the output signals in the time domain are obtained. As opposed to the circuits with constant parameters, in the circuits with variable parameters the coefficients of the numerator of the used decompositions become dependent on time. Thus, if the image of the output signal of the linear parametric circuit can be represented in the form

$$Y(s,t) = W(s,t) \cdot X(s) = \frac{P_1(s,t) \cdot P_2(s)}{Q_1(s) \cdot Q_2(s)}, \qquad (7)$$

then [7]:

$$y(t) = \sum_{m=1}^{n} \frac{P_1(s_m,t) \cdot P_2(s_m)}{\frac{d}{ds}[Q_1(s) \cdot Q_2(s)]_{s=s_m}} \cdot e^{s_m \cdot t}, \qquad (8)$$

where $\hat{W}(s,t) = \frac{P_1(s,t)}{Q_1(s)}$, $X(s) = \frac{P_2(s)}{Q_2(s)}$, s_m are the roots of the equations $Q_1(s) = 0$ and $Q_2(s) = 0$.

It is exactly expression (8) which is to be used in the calculations performed below. However, it requires calculating the roots of the denominator polynomials and so should be used with certain caution.

3. Computer experiments

Experiment 1. Using the FS-method, the authors performed the analysis of the steady-state mode of the single-circuit parametric amplifier in the time domain at the simultaneous change of two parametric elements $c(t)$ and $L(t)$ shown in Fig. 1.

The tasks of the experiment were as follows:

– assume that the input signal is the harmonious one and can be represented in the form $i(t) = I_m \cdot \cos(w_c \cdot t + j), I_m = 0.0001A,\ j = -p/4$;

– use the approximation $\hat{W}(s,t)$ of the conjugate parametric transfer function $W(s,t) = U_2(s,t)/I(s)$, where $s = jw$, $I(s)$ and $U_2(s,t)$ are the images of the input $i(t)$ and output $u_2(t)$ signals, respectively;

– applying expression (8) and the system of software functions MAOPCs, draw the graph of output voltage $u_2(t)$ at changing the phase difference $(j_c - j_L)$ of the parametric elements from 0° to 180°;

– applying expression (8) and the system of software functions MAOPCs, calculate the instantaneous value of the output voltage u_2(t) at $t = 0.800 \cdot 10^{-6}:0.001 \cdot 10^{-6}:0.805 \cdot 10^{-6}$ s and the change of the phase difference $(j_c - j_L)$ from 0° to 180°;

– compare the obtained value of output voltage with the values received by the Micro-Cap 7.0 program [7] which are generally accepted to be adequate.

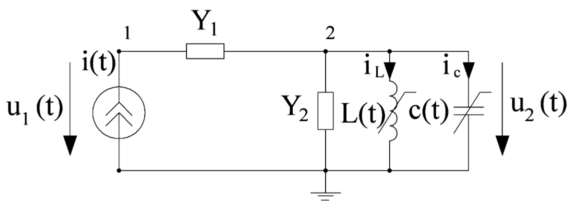

Fig. 1. Single-circuit parametric amplifier with two parametric elements - $c(t) = c_0 \cdot (1 + m_c \cdot \cos(\Omega \cdot t + j_c))$,

$c_0 = 10 \cdot 10^{-12} F,\ L(t) = L_0 \cdot (1 + m_L \cdot \cos(\Omega \cdot t + j_L))$,

$L_0 = 0.2533 \cdot 10^{-6} H,\ Y_1 = 0.25S,\ Y_2 = 0.0004S$,

$m_c = 0.01, m_L = 0.1, \Omega = 4 \cdot p \cdot 10^8\, rad/s$.

The results of the experiment are as follows.

1. The image of the input signal according to the Fourier transform is the following

$$I_{in}(s) = \frac{\sqrt{2} \cdot 10^{-4} \cdot (s + 2 \cdot p \cdot 10^8)}{2 \cdot (s^2 + (2p \cdot 10^8)^2)}.$$

2. The image of the output signal in the form of (7) is determined by the functions $P_1(s,t)$ and $Q_1(s)$, obtained by the system of software functions MAOPCs at sufficient value $k = 6$. Due to the intricacy of the specified functions, they are shown here with the value $k = 1$:

$$P_1(s,t) = 2.0270 \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) +$$
$$+0.4827 \cdot 10^{-7} \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s -$$
$$-0.8118 \cdot 10^{-44} \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^5 +$$
$$+0.7757 \cdot 10^{-18} \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^2 +$$
$$+0.4827 \cdot 10^{-7} \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s -$$
$$-0.8118 \cdot 10^{-44} \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^5 -$$
$$-0.3265 \cdot 10^{-36} \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^4 -$$
$$-0.3265 \cdot 10^{-36} \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^4 +$$
$$+0.3395 \cdot 10^{-26} \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^3 +$$
$$+0.3395 \cdot 10^{-26} \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^3 +$$
$$+0.7757 \cdot 10^{-18} \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^2 +$$
$$+47.7900 \cdot j \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right) -$$
$$-47.7900 \cdot j \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) +$$
$$+0.9028 \cdot 10^{-37} \cdot j \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^4 -$$
$$-0.6412 \cdot 10^{-9} \cdot j \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s -$$
$$-0.9028 \cdot 10^{-37} \cdot j \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^4 -$$
$$-0.3991 \cdot 10^{-27} \cdot j \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^3 -$$
$$-0.4834 \cdot 10^{-16} \cdot j \cdot exp\left(0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^2 +$$
$$+0.3991 \cdot 10^{-27} \cdot j \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^3 +$$
$$+0.6412 \cdot 10^{-9} \cdot j \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s +$$
$$+0.4834 \cdot 10^{-16} \cdot j \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right) \cdot s^2 +$$
$$+0.1294 \cdot 10^{-33} \cdot s^4 + .02555 \cdot 10^{-15} \cdot s^2 +$$
$$+0.1618 \cdot 10^{-41} \cdot s^5 + 0.6393 \cdot 10^{-23} \cdot s^3 +$$
$$+0.2272 \cdot 10^{-5} \cdot s + 2.0270 \cdot exp\left(-0.1257 \cdot 10^{10} \cdot j \cdot t\right),$$

$$Q_1(s) = 0.1922 \cdot 10^{-8} \cdot s + 0.7040 \cdot 10^{-34} \cdot s^4 +$$
$$+0.4818 \cdot 10^{-16} \cdot s^2 + 0.1617 \cdot 10^{-52} \cdot s^6 +$$
$$+0.1940 \cdot 10^{-44} \cdot s^5 + 9.0280 + 0.5625 \cdot 10^{-26} \cdot s^3, \quad (9)$$

and by functions $P_2(s)$ and $Q_2(s)$

$$P_2(s) = 10^{-4} \cdot (s \cdot \cos(-p/4) - 2p \cdot 10^8 \cdot \sin(-p/4)),$$
$$Q_2(s) = s^2 + (2p \cdot 10^8)^2. \quad (10)$$

Expressions (9) and (10) are the basis for calculations of instantaneous values of the output voltage of the amplifier in the steady state with the use of the the system of software functions MAOPCs and expression (8):

– Fig. 2 shows the obtained time dependence of the output voltage for different given values of the phase difference $(j_c - j_L)$;

– Table 1 contains the numerical values of the output voltage of the amplifier obtained at the given time points and different given values of the phase difference $(j_c - j_L)$.

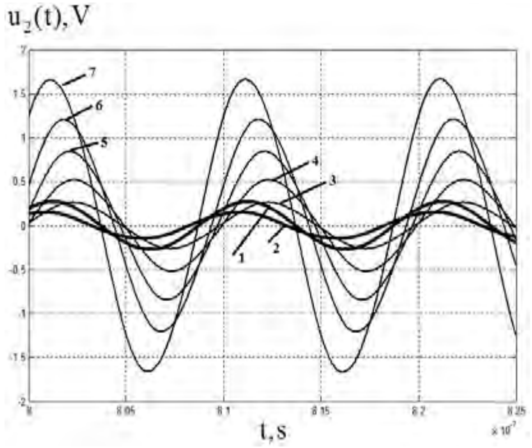

Fig. 2. Time dependence of the output voltage $u_2(t)$ of the amplifier depicted in Fig. 1, obtained by the system of software functions MAOPCs for two cases: 1 – one parametric element in the circuit $c(t)$, $L = 0.2533 \cdot 10^{-6} H$; 2 – two parametric elements in the circuit $c(t)$ and $L(t)$ $(j_c - j_L) = 0^0$; 3 – $(j_c - j_L) = 30^0$; 4 – $(j_c - j_L) = 60^0$; 5 – $(j_c - j_L) = 90^0$; 6 – $(j_c - j_L) = 120^0$; 7 – $(j_c - j_L) = 180^0$

The coincidence of the 4th and 5th significant figures of numeric values of the output voltages from Table 1 obtained via the system of software functions MAOPCs and the Micro-Cap 7.0 program demonstrates the adequacy of results obtained by the FS-method.

Table 1

Instantaneous values of the voltage $u_2(t)$

t, μs	0.800	0.801	0.802	0.803
1	2	3	4	5
$(j_c - j_L) = 0^0$				
$u_2(t), V$ by Micro-Cap	0.12908	0.14815	0.11201	0.02445
$u_2(t), V$ by MAOPCs	0.12908	0.14815	0.11201	0.02445
$(j_c - j_L) = 30^0$				
$u_2(t), V$ by Micro-Cap	0.02899	0.17156	0.25789	0.23296
$u_2(t), V$ by MAOPCs	0.02894	0.17159	0.25784	0.23296
$(j_c - j_L) = 60^0$				
$u_2(t), V$ by Micro-Cap	0.02685	0.32372	0.50730	0.47107
$u_2(t), V$ by MAOPCs	0.02681	0.32375	0.50732	0.47108
$(j_c - j_L) = 90^0$				
$u_2(t), V$ by Micro-Cap	0.15617	0.62084	0.84094	0.70826
$u_2(t), V$ by MAOPCs	0.15610	0.62085	0.84090	0.70822
$(j_c - j_L) = 120^0$				
$u_2(t), V$ by Micro-Cap	0.44794	1.03862	1.19504	0.88196
$u_2(t), V$ by MAOPCs	0.44791	1.03860	1.19507	0.88196
$(j_c - j_L) = 180^0$				
$u_2(t), V$ by Micro-Cap	1.23409	1.65232	1.41933	1.69001
$u_2(t), V$ by MAOPCs	1.23401	1.65239	1.41939	0.69003

Experiment 2. Using the FS-method, the time analysis of the transition mode of the single-circuit parametric amplifier depicted in Fig. 1 was performed.

The tasks of the experiment were as follows:

– assume that an input signal is the Heaviside step function $i(t) = 1(t)$;

– by analogy with example 1, use the approximation $\hat{W}(s,t)$ of the conjugate parametric transfer function $W(s,t) = U_2(s,t)/I(s)$, where $s = s + jw$;

– using expression (8) and the system of software functions MAOPCs, draw graphical dependence of the output voltage $u_2(t)$ at t = 0:0.000001·10^-6: :0.25·10^-6 s, the change of modulation depth m_c of the parametric capacity from 0.03 to 0.27 and at $j_c = 0^0, j_L = 0^0, m_L = 0.1$;

– using expression (8) and the the system of software functions MAOPCs, calculate the instantaneous value of the output voltage $u_2(t)$ at $t = 0.1 \cdot 10^{-8}:0.1 \cdot 10^{-8}:0.5 \cdot 10^{-8}$ s, the change of modulation depth m_c of the parametric capacity from 0.03 to 0.27 and at $j_c = 0^0, j_L = 0^0, m_L = 0.1$;

– compare the obtained value of output voltage with the values received via the Micro-Cap 7.0 program which are considered to be adequate.

The results of the experiment are as follows.

1. The image of the input signal according to the Laplace transform is determined by an expression $I_{in}(s) = 1/s$.

2. The image of the output signal in the form of (7) has the same functions $P_1(s,t)$, $Q_1(s)$ from (9) for $k = 6$ and $P_2(s) = 1$, $Q_2(s) = s$.

Expressions (9) by analogy with the experiment 1 are also the basis for calculations of instanteous values of the output voltage of the amplifier in the transition mode by the expression (8) via the system of software functions MAOPCs;

– Fig. 3 shows the obtained time dependencies of the output voltage for various given values of depth modulation m_c of the parametric capacity;

– Table 2 contains the numerical values of the output voltage of the amplifier obtained for the given time points and for various preset values m_c .

The coincidence of the 4th and 5th significant figures of numeric values of the output voltages in Table 2 obtained via the system of software functions MAOPCs and Micro-Cap program, by analogy with experiment 1, also demonstrates the adequacy of the received FS-method results. It is necessary to note an interesting phenomenon which can be observed in the time range $t = (2.0 \div 2.5) \cdot 10^{-7} s$ of the transients depicted in Fig. 3, a–g: when m_c rises from 0,03 to 0,09 (Fig. 3, a–c) the amplitude of the signal decreases; then, when m_c increases from 0.09 to 0.21 (Fig. 3, c–f), signal amplitude increases. Finally, at the value $m_c = 0.27$, as derived from Fig. 3, g, the amplifier becomes unstable (Fig. 4). This peculiar phenomenon was considered in [9], while analysing the stability of the amplifier depicted in Fig. 1.

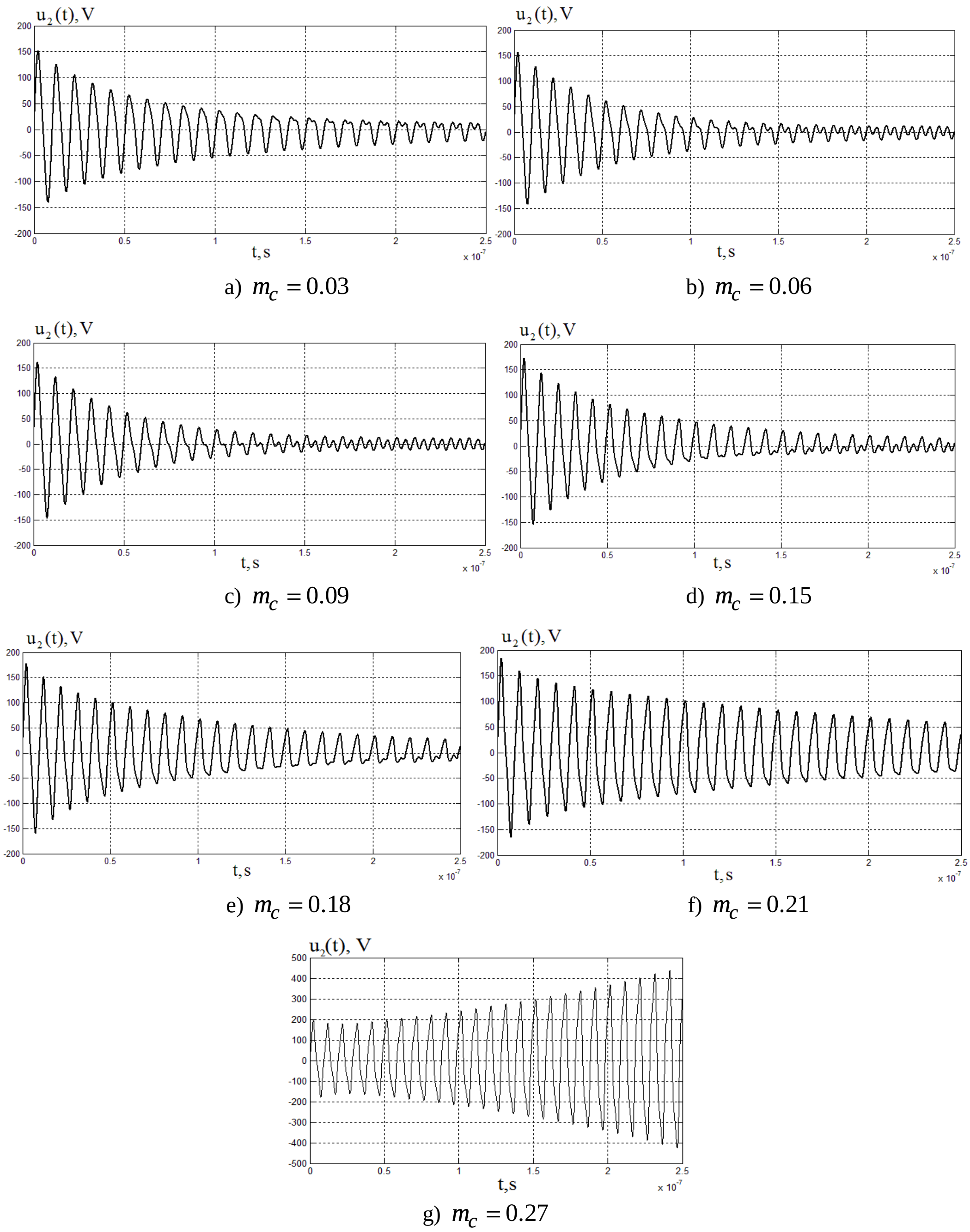

Fig. 3. Time dependence of the output voltage $u_2(t)$ of the amplifier, depicted in Fig. 1, in a transition mode, feeding the input Heaviside step function and for different values of modulation depth m_c.

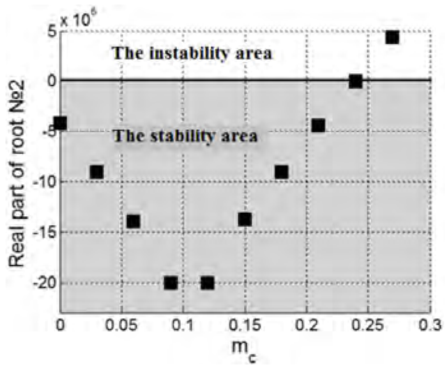

Fig.4. The map of the stability margin of the single-circuit parametric amplifier at $j_c = 0^{\mathbf{0}}, j_L = 0^{\mathbf{0}}, k = 6, m_L = 0.1$

Table 2

Instantaneous values of the voltage u_2(t)

t ,μs	0.001	0.002	0.003	0.004
1	2	3	4	5
$m_c = 0.03$				
$u_2(t), V$ by Micro-Cap	91.3542	147.8293	134.4481	73.4786
1	2	3	4	5
$u_2(t), V$ by MAOPCs	91.3542	147.8293	134.4481	73.4786
$m_c = 0.06$				
$u_2(t), V$ by Micro-Cap	90.6804	151.8875	136.6407	69.7815
$u_2(t), V$ by MAOPCs	90.6804	151.8875	136.6407	69.7815
$m_c = 0.09$				
$u_2(t), V$ by Micro-Cap	90.0121	156.1322	138.8270	65.9691
$u_2(t), V$ by MAOPCs	90.0121	156.1322	138.8270	65.9691
$m_c = 0.15$				
$u_2(t), V$ by Micro-Cap	88.6930	165.2459	143.1539	57.9725
$u_2(t), V$ by MAOPCs	88.6930	165.2459	143.1549	57.9725
$m_c = 0.18$				
$u_2(t), V$ by Micro-Cap	88.0427	170.1516	145.2766	53.7732
$u_2(t), V$ by MAOPCs	88.0427	170.1516	145.2766	53.7732
$m_c = 0.21$				
$u_2(t), V$ by Micro-Cap	87.3988	175.3177	147.3566	49.4279
$u_2(t), V$ by MAOPCs	87.3988	175.3177	147.3566	49.4279
$m_c = 0.27$				
$u_2(t), V$ by Micro-Cap	86.1305	186.5294	151.3230	40.2559
$u_2(t), V$ by MAOPCs	86.1305	186.5294	151.3230	40.2559

Experiment 3. With the use of the FS-method, the time analysis of the steady-state mode of the double-circuit parametric amplifier with one parametric element $c(t)$, shown in Fig. 5, was performed.

The tasks of the experiment were as follows:

– assume that the input signal is harmonious, its form being $i(t) = I_m \cdot \cos(w_c \cdot t + j), I_m = 0.1mA,\ j = p/4$;

– use the approximation $\hat{W}(s,t)$ of the conjugate parametric transfer function $W(s,t) = U_C(s,t)/I(s)$ **,** where $s = jw$, $I(s)$ and $U_C(s,t)$ are the images of the input signal $i(t)$ and parametric capacity voltage $u_C(t)$;

– using expression (8) and the system of software functions MAOPCs, calculate the instantaneous value of the voltage $u_C(t)$ at $t = 70 \cdot 10^{-6} : 10^{-9} : 70.007 \cdot 10^{-6}$ s;

– compare the obtained voltage values $u_C(t)$ with the values obtained by the Micro-Cap 7.0 program [7] which are considered to be adequate.

The results of the experiment are as follow.

1. The image of the input signal according to the Fourier transform is the following

$$I(s) = \frac{\sqrt{2} \cdot 10^{-4} \cdot (s - 2 \cdot p \cdot 10^8)}{2 \cdot (s^2 + (2p \cdot 10^8)^2)}. \qquad (11)$$

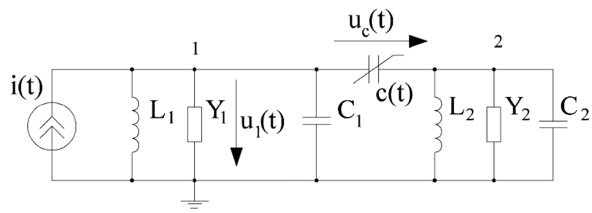

Fig. 5. Double-circuit parametric amplifier

$c(t) = c_0 \cdot (1 + m \cdot \cos(\Omega \cdot t)),$

$c_0 = 1pF; m = 0.1; \Omega = 2 \cdot p \cdot 298.573 \cdot 10^6 rad/s;$

$s = jw_c; w_c = 2 \cdot p \cdot 10^8 rad/s; Y_1 = Y_2 = 0.0001S;$

$C_1 = C_2 = 68pF; L_1 = 36.70795nH; L_2 = 9.312609nH.$

2. The image of the output signal in the form of (7) is determined by the functions $P_1(s,t)$ and $Q_1(s)$ obtained via the system of software functions MAOPCs and sufficient value $k = 2$. Due to the intricacy of the specified functions, they are shown here with the value $k = 1$:

$$P_1(s,t) = (473 \cdot 10^{13} \cdot s^2 + 695 \cdot 10^{19} \cdot s + 746 \cdot 10^{31}) \times$$
$$\times (-s) \cdot \left(116 \cdot 10^{71} \cdot exp\left(196 \cdot 10^{13} / 104 \cdot 10^4 \cdot j \cdot t\right) \cdot s^5 + \right.$$
$$+ 334 \cdot 10^{107} \cdot exp\left(-196 \cdot 10^{13} / 104 \cdot 10^4 \cdot j \cdot t\right) \cdot s$$

$$-279\cdot 10^{110}\cdot s-402\cdot 10^{92}\cdot s^3-192\cdot 10^{104}\cdot s^2-$$
$$-138\cdot 10^{86}\cdot s^4-865\cdot 10^{54}\cdot s^7-116\cdot 10^{74}\cdot s^5-$$
$$-149\cdot 10^{48}\cdot s^8-268\cdot 10^{67}\cdot s^6-561\cdot 10^{121}+$$
$$+116\cdot 10^{119}\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)+$$
$$+116\cdot 10^{119}\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)+$$
$$+313\cdot 10^{98}\cdot j\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^2+$$
$$+942\cdot 10^{79}\cdot j\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^4+$$
$$+639\cdot 10^{91}\cdot j\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^3+$$
$$+135\cdot 10^{119}\cdot j\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)-$$
$$-942\cdot 10^{79}\cdot j\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^4-$$
$$-639\cdot 10^{91}\cdot j\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^3-$$
$$-313\cdot 10^{98}\cdot j\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^2-$$
$$-349\cdot 10^{39}\cdot j\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^7+$$
$$+655\cdot 10^{109}\cdot j\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s+$$
$$+769\cdot 10^{72}\cdot j\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^5$$
$$+571\cdot 10^{60}\cdot j\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^6+$$
$$+349\cdot 10^{39}\cdot j\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^7-$$
$$-769\cdot 10^{72}\cdot j\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^5-$$
$$-655\cdot 10^{109}\cdot j\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s-$$
$$-571\cdot 10^{60}\cdot j\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^6+$$
$$+185\cdot 10^{83}\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^4+$$
$$+362\cdot 10^{64}\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^6+$$
$$+296\cdot 10^{101}\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^2+$$
$$+397\cdot 10^{89}\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^3+$$
$$+116\cdot 10^{71}\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^5+$$
$$+296\cdot 10^{101}\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^2+$$
$$+397\cdot 10^{89}\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^3+$$
$$+334\cdot 10^{107}\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s+$$
$$+185\cdot 10^{83}\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^4+$$
$$+362\cdot 10^{64}\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^6+$$
$$+931\cdot 10^{51}\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)s^7+$$
$$+213\cdot 10^{45}\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)s^8+$$
$$+931\cdot 10^{51}\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^7+$$
$$+213\cdot 10^{45}\cdot exp\left(196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\cdot s^8-$$
$$-135\cdot 10^{116}\cdot j\cdot exp\left(-196\cdot 10^{13}/104\cdot 10^4\cdot j\cdot t\right)\Big),$$

$$Q_1(s)=171\cdot 10^{128}\cdot s^4+109\cdot 10^{152}\cdot s+368\cdot 10^{98}\cdot s^7+$$
$$+691\cdot 10^{116}\cdot s^5+636\cdot 10^{91}\cdot s^8+497\cdot 10^{134}\cdot s^3+$$
$$+755\cdot 10^{145}\cdot s^2+159\cdot 10^{110}\cdot s^6+984\cdot 10^{72}\cdot s^{10}+$$
$$+713\cdot 10^{79}\cdot s^9+114\cdot 10^{169}+429\cdot 10^{59}\cdot s^{11}+$$
$$+494\cdot 10^{53}\cdot s^{12}. \qquad (12)$$

and by the functions $P_2(s)$ and $Q_2(s)$ in the form:

$$P_2(s)=10^{-4}\cdot(s\cdot\cos((p/4)-w\cdot\sin(p/4)),$$
$$Q_2(s)=s^2+(2\cdot p\cdot 10^8)^2. \qquad (13)$$

Expressions (12) and (13) are the basis for calculations of instanteous values of the voltage $u_c(t)$ of the amplifier in the steady state with the use of expression (8) and the system of software functions MAOPCs:

– Figure 6 shows the obtained time dependence of the voltage $u_C(t)$;

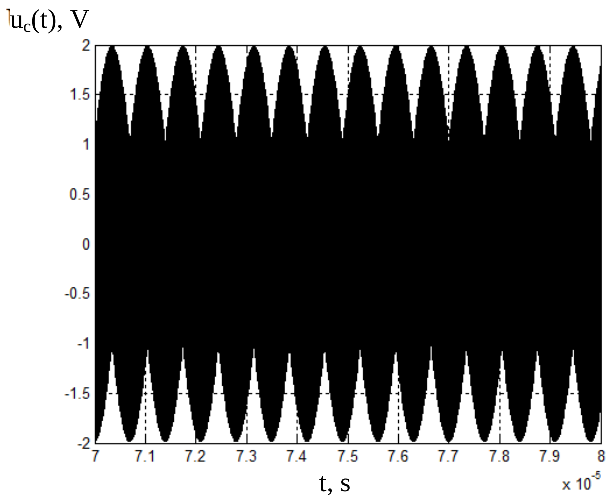

Fig. 6. Time dependence of the output voltage $u_C(t)$ of the amplifier depicted in Fig.4, obtained via the system of software functions MAOPCs

– Table 3 shows the numerical values of the voltage $u_C(t)$ of the amplifier obtained for the given time points.

Table 3

Instantaneous values of voltage $u_C(t)$

t ,μs	$u_c(t),V$ by Micro-Cap	$u_c(t),V$ by MAOPCs	
		k=1	k=2
1	2	3	4
70000	0.807	0.807	0.807
70001	0.876	0.875	0.876
70002	-0.065	-0.064	-0.065
70003	-1.466	-1.466	-1.466
70004	-1.958	-1.957	-1.958
70005	-0.978	-0.979	-0.978
70006	0.473	0.474	0.473
70007	1.087	1.087	1.087
70008	0.773	0.775	0.773

Experiment 4. Using the FS-method, the time analysis of the transition mode of the double-circuit parametric amplifier depicted in Fig. 5 was performed.

The tasks of the experiment were as follows:

– assume that an input signal is the Heaviside step function $i(t)=1(t)$;

– by analogy with experiment 3, use the approximation $\hat{W}(s,t)$ of the conjugate parametric transfer function $W(s,t)=U_C(s,t)/I(s)$, where $s=s+jw$;

– using expression (8) and the system of software functions MAOPCs, draw the graphical dependence of the voltage $u_C(t)$ at $\mathrm{t}=0:0.7\cdot10^{-6}$;

– using expression (8) and the system of software functions MAOPCs, calculate the instantaneous value of the voltage $u_C(t)$ at $t=0.1\cdot10^{-6}:0.1\cdot10^{-6}:0.7\cdot10^{-6}$ s;

– compare the obtained voltage values $u_C(t)$ with the values obtained via the Micro-Cap 7.0 program, the latter being assumed to be adequate.

The results of the experiment are shown below.

1. The image of the input signal according to the Laplace transform is the expression $I_{in}(s)=1/s$.

2. The image of the output signal in the form of (7) has the same functions $P_1(s,t)$, $Q_1(s)$ from (12) and $P_2(s)=1$, $Q_2(s)=s$.

Expressions (12) by analogy with experiment 3 are also the basis for calculating the instanteous values of the voltage $u_C(t)$ of the amplifier in the transition mode by expression (8) and via the system of software functions MAOPCs:

– Fig. 7 shows the obtained time dependencies of the voltage $u_C(t)$;

– Table 4 shows the numerical values of the voltage $u_C(t)$ of the amplifier obtained at the given time points.

The coincidence of the 3rd and 4th significant figures of numeric values of the voltages $u_C(t)$ from Table 4 obtained via the system of software functions MAOPCs and Micro-Cap program, by analogy with experiment 3, also demonstrates the adequacy of the results obtained by the FS-method.

Table 4

Instantaneous values of voltage $u_C(t)$

t ,μs	$u_C(t),V$ by Micro-Cap	$u_C(t),V$ by MAOPCs	
		k=1	k=2
0.1	0.7413	0.7410	0.7413
0.2	-0.2015	-0.2041	-0.2015
0.3	-2.1240	-2.1217	-2.1240
0.4	-2.8182	-2.8149	-2.8182
0.5	-0.9670	-0.9713	-0.9670
0.6	2.3007	2.3001	2.3007
0.7	4.2531	4.2603	4.2531

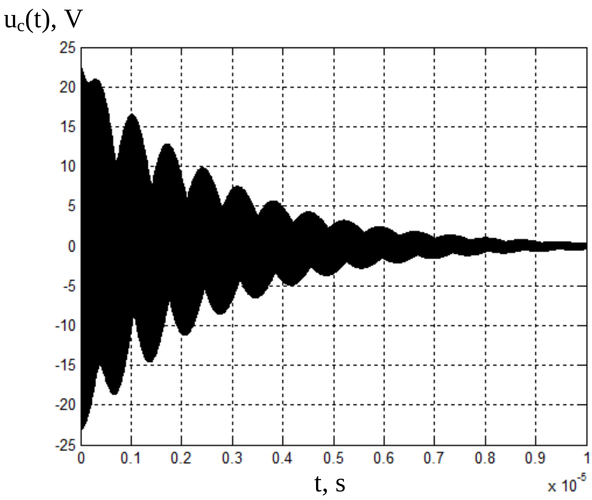

Fig. 7. Time dependence of the output voltage $u_C(t)$ of the amplifier depicted in Fig.4 in a transition mode, when feeding input Heaviside step function.

4. Conclusions

The coincidence of the results obtained by the system of software functions MAOPCs and the Micro-Cap7.0 program demonstrates the adequacy of applying the inverse Fourier and Laplace transforms for the investigations of LPTV circuits in the steady and transient modes in an environment of the MAOPCs.

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ЗАСТОСУВАННЯ ЧАСТОТНОГО СИМВОЛЬНОГО МЕТОДУ ДЛЯ АНАЛІЗУ ЛІНІЙНИХ ПАРАМЕТРИЧНИХ КІЛ У ЧАСОВІЙ ОБЛАСТІ

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З’ясовано проблему застосування частотного символьного методу до визначення часових залежностей вихідних сигналів лінійних параметричних кіл на підставі застосування оберненого перетворення Фур’є (для усталеного режиму) чи Лапласа (для перехідного режиму) до зображення цих сигналів отриманих з використанням передавальних функцій. Частотний символьний метод дає можливість обчислювати спряжені параметричні передавальні функції лінійних параметричних кіл, що зв’язують вхідні сигнали з вихідними у вигляді апроксимуючих поліномів Фур’є у тригонометричній чи комплексній формі. Такими ж поліномами Фур’є апроксимуються нормальні параметричні передавальні функції, які є основою оцінювання асимптотичної стійкості кола. Для визначення спряжених та нормальних передавальних функцій ми використовуємо систему програмних функцій MAOPCs (Multivariate Analysis and Optimization of the Parametric Circuits), передбачено відповідні програмні функції. Система програмних функцій MAOPCs основана на частотному символьному методі.

У роботі наведено результати обчислювальних експериментів, отриманих за системою програмних функцій MAOPCs, що показують адекватність визначення усталених та перехідних режимів лінійних параметричних кіл у часовій області за допомогою спряжених передавальних функцій, визначених за частотним символьним методом.

Збіг результатів, отриманих за допомогою системи програмних функцій MAOPCs та за програмою Micro-Cap7.0, свідчить про адекватність застосування оберненого перетворення Фур’є та Лапласа для дослідження лінійних параметричних кіл в усталеному та перехідному режимах у середовищі MAOPCs.

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