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# THE ALGORITHM OF IMAGES CENTERING CONSTRUCTED ON THE BASIS OF COMBINATION OF THE TOTAL COUNT AND CORRELATION ANALYSIS METHODS 

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The algorithm of images centering, based on the successive application of the mathematical statistics and total count methods, has been improved. The method is based on the analysis of the color vector flux through the image surface with the subsequent elaboration of the results obtained by the correlation analysis methods.
Keywords - correlation, integral, resolution, image.

## 1. Introduction

The effectiveness of the image digital processing increases significantly due to the development of quickacting digital conversion algorithms for many applications solving.

Among the methods of image recognition and analysis, the methods consisting in the comparison of images of the object being identified with the pattern and in the computation of the function of correlation between 2-D functions of the compared images brightness, are known [1-3].

Most of the image analysis systems based on patterns are directed to the object recognition when the processing and the structural analysis are separately performed for each object image. It is the so-called directed approach.

There is a parallel approach when the processing and the analysis include all objects found during a line (column) of the image matrix scanning. The registration of objects relation during the structural analysis provides unique information for their identification. It is quite difficult to obtain such information using the directed approach. The parallel approach allows to reveal and to classify this relation in the normal fashion during the serial scanning of the matrix image scene: from left to right and top-down [3].

One of the basic ways to find objects is to search for them on an image and to recognize them using the method of comparison with a pattern [5]. The correlation between the processed image and the pattern is used as the estimation value. The formal essence of correlation processing lies in the calculation of the 2-D matrix-vector product and in the analysis of the obtained correlation matrix [6]. The main disadvantage of the correlation method of similarity estimation is a great number of calculations; hence is a small speed of the method.

One of the informative classes of images analyzed in this paper is the halftone class, i.e. the one in which the numbers of quite a narrow range are the matrix elements. In practice, the most optimal images are the ones with 256 levels of brightness [4].

The object of this paper is to solve the problem of image centering within the given $\Delta$ value on the basis of combination of the mathematical statistics and total count methods.

The basic task is to construct an algorithm that would allow centering the image within the given $\Delta$ value with the minimum time expenditure. The primary bases for this problem solving are the total count methods.

## 2. Problem statement

Let there be $t$, a measuring set of objects shifted on the geometrical surface,

$$
\begin{equation*}
\mathrm{P}=\{\mathrm{Pn}\}, \quad \mathrm{n}=1 . . \mathrm{t}, \tag{1}
\end{equation*}
$$

where $P$ is the nth image from the $t$-dimensional set, $n$ is the number of image from the set and $t$ is the quantity of images in the set.

Such set can be obtained in different ways, e.g. by a repeated delayed photography or video filming of an object.

These sets are formed in many ways: by moving a camera, an object etc. The shift magnitude can exceed the previously specified $\Delta$ value which generally is the Cartesian coordinates function, i.e. $\Delta=\Delta(x, y, z)$ and in most
cases is the basis for the mathematical methods of subsequent images processing. In that case, a problem of centering the images within the specified $\Delta$ value arises.

## 3. The mathematical model

The method offered solves the problem of centering within the specified $\Delta$ value. Today it provides vertical and horizontal centering within one pixel.

When actually received, the set of images $\{\mathrm{Pn}\}, \mathrm{n}=1$..t can be very noisy. Therefore, this set should be preliminary filtered. Such filtration can be done according to the procedure described in [8]. As a result of functioning of the relative $\mathrm{P}=\{\mathrm{Pn}\}, \mathrm{n}=1 . . \mathrm{t}$ filtration algorithm [8] we receive the set $\Lambda=\{\Lambda \theta\}, \theta=1 . . \Theta$, such as

$$
\begin{equation*}
\forall \theta \in[1 . . \Theta], \quad \exists \mathrm{n} \in[1 . . \mathrm{t}]: \quad \Lambda \theta=\mathrm{Pn} . \tag{2}
\end{equation*}
$$

That is, $\{\Lambda \theta\}$ is a subset of the $\{\mathrm{Pn}\}$ set, and the following inequality takes place:
$\Theta \leq \mathrm{t}$.
Hereinafter the images from the subset $\{\Lambda \theta\}, \quad \theta=1 . . \Theta$ are considered.
A fragment of the $\Lambda \theta$ image of a given magnitude, i.e. with the height h and width 1 , is considered to be the frame. The set of frames forms the covering of the $\Lambda \theta$ image

$$
\begin{equation*}
\Lambda_{\theta}=\bigcup_{m=1}^{d_{\theta}} F_{m}^{\Lambda_{\theta}}, \quad \theta=1 . . \Theta \tag{4}
\end{equation*}
$$

where $\mathrm{d} \theta$ is the number of the $\Lambda \theta$ image frames. To simplify the solving of the problem posed, all the frames are hereinafter considered to have equal h and l .

Generally, the following dependence can be considered

$$
\begin{equation*}
F_{m}^{\Lambda_{\theta}}=\mathrm{F}(\Lambda \theta, \mathrm{~m}, \mathrm{~h}, \mathrm{l}), \quad \theta=1 . . \Theta, \quad \mathrm{m}=1 . . d_{\theta}, \tag{5}
\end{equation*}
$$

where m is the frame number in the $\Lambda \theta$ image.
The flux of the vector $\mathbf{Q}_{\theta}=\left(Q_{1}, \ldots, Q_{N}\right)$, as the flux of the color vector through the $\Lambda \theta$ surface, is the basis of the mathematical model of image centering, i.e. the following is considered

$$
\begin{equation*}
\Upsilon_{\theta}=\int_{\Lambda_{\theta}} \mathbf{Q}_{\theta} d \Lambda_{\theta}, \quad \theta=1 . . \Theta \tag{6}
\end{equation*}
$$

The variable N defines the range of the permissible values of color in the given palette. Thus, for the gray color gradation scale we have $\mathrm{N}=255$.

Taking into account (4) and the integral additivity [7], we can write the following

$$
\begin{equation*}
\Upsilon_{\theta} \leq \sum_{m}^{d_{\theta}} \int_{F_{m}^{\Lambda_{\theta}}} \mathbf{Q}_{m}^{\theta} d F_{m}^{\Lambda_{\theta}}, \quad \theta=1 . . \Theta \tag{7}
\end{equation*}
$$

where $\mathbf{Q}_{m}^{\theta}$ is the flux of the $\mathrm{Q} \theta$ vector through the $F_{m}^{\Lambda_{\theta}}$ frame.
If to introduce the designation

$$
\begin{equation*}
\Upsilon_{m}^{\theta}=\int_{F_{m}^{\Lambda_{\theta}}} \mathbf{Q}_{m}^{\theta} d F_{m}^{\Lambda_{\theta}}, \quad \theta=1 . . \Theta . \tag{8}
\end{equation*}
$$

then (7) can be represented as

$$
\begin{equation*}
\Upsilon_{\theta} \leq \sum_{m}^{d_{\theta}} \Upsilon_{m}^{\theta}, \quad \theta=1 . . \Theta \tag{9}
\end{equation*}
$$

Each $F_{m}^{\Lambda_{\theta}}$ frame is characterized by a discrete set of colors, i.e. the frame characteristics

$$
\begin{equation*}
Q_{m}^{\Lambda_{\theta}}=Q_{m}^{d_{\theta}}(i, j), \quad \theta=1 . . \Theta, \quad m=1 . . d_{\theta} \tag{10}
\end{equation*}
$$

where $Q_{i, j}$ is the color value in the point with the $(i, j)$ coordinates of the mth frame, as well as with the geometrical coordinates of shift within the $\Lambda \theta$ image.

$$
\left\{\begin{array}{l}
x_{\text {поч }}^{F_{M_{\theta}}^{\Lambda_{\theta}}}=x\left(F_{m}^{\Lambda_{\theta}}\right) ;  \tag{11}\\
y_{\text {поч }}^{F_{\theta}}=y\left(F_{m}^{\Lambda_{\theta}}\right) ;
\end{array} \quad \theta=1 . . \Theta\right.
$$

Here $x_{\text {поч }}^{F_{m}^{\Lambda_{\theta}}}$ i $y_{\text {поч }}^{F_{m}^{\Lambda_{\theta}}}$ define the coordinates of the left upper corner of the $F_{m}^{\Lambda_{\theta}}$ frame.
Thus, each frame can be compared to a set which uniquely defines it

$$
\begin{equation*}
F_{m}^{\Lambda_{\theta}} \rightarrow\left[x_{\text {поч }}^{F_{m}^{\Lambda_{\theta}}}, y_{\text {поч }}^{F_{m}^{\Lambda_{\theta}}}, Q_{m}^{\Lambda_{i, j}} \Lambda_{\theta}\right] . \tag{12}
\end{equation*}
$$

Taking into account (10), the integral (8) can be brought to the double [7]

The basis of the method offered is the finding, with respect to the second specified frame (e.g. on the $\Lambda 1$ image, i.e. $F_{2}^{\Lambda_{1}}$ ), of all frames on the $\{\Lambda \theta\}, \theta=2 . . \Theta$ images for which the condition of similarity is fulfilled.

$$
\begin{equation*}
\forall \theta \in[2 . . \Theta] \wedge \forall m \in\left[1 . . d_{\theta}\right]: \quad\left\|\Upsilon_{2}^{1}-\Upsilon_{m}^{\theta}\right\|=\sqrt{\left(\int_{F_{2}^{\Lambda_{1}}} \mathbf{Q}_{2}^{1} d F_{2}^{\Lambda_{1}}-\int_{F_{m}^{\Lambda_{\theta}}} \mathbf{Q}_{m}^{\theta} d F_{m}^{\Lambda_{\theta}}\right)^{2}}<\varepsilon \tag{14}
\end{equation*}
$$

Here $\varepsilon$ is the error value defined in the following way:

$$
\begin{equation*}
\varepsilon=k \Lambda_{1}^{F_{2}^{\Lambda_{1}}}, k=1 . .10 \% \tag{15}
\end{equation*}
$$

As a result of the condition (14) action, the following set of frames will be obtained:

$$
\begin{equation*}
\left\{{ }_{m}^{I_{\theta}}\right\}, \mathrm{l}=1 . . \mathrm{k}, \tag{16}
\end{equation*}
$$

where k is the number of frames (set dimension). The $\theta=2 . . \Theta$ and $\mathrm{m}=1 . . \mathrm{d} \theta$ variables denote only the unique identification of the frame. E.g. if $\theta=2$ and $m=3$, we have

$$
I_{3 l}^{\Lambda_{2}}=F_{3}^{\Lambda_{2}}
$$

The obtained $\left\{\begin{array}{c}\left.I_{m}^{\Lambda_{\theta}}{ }^{l}\right\} \text { is a set of frames which, according to the condition (14) with accuracy (15), are }\end{array}\right.$ similar to the previously specified $F_{2}^{\Lambda_{1}}$ frame. This set should be defined more exactly since several frames from the set (16) can relate to one $\Lambda \theta$; it is proposed to make such definition by searching for the correlation dependence

$$
\begin{equation*}
\rho_{l}=\frac{\operatorname{cov}\left(F_{2}^{\Lambda_{1}}, I_{m l}^{\Lambda_{\theta}}\right)}{\sigma_{F_{2}^{\Lambda_{1}}} \sigma_{I_{m^{\prime}}^{\Lambda_{0}}}} ; \quad l=1 . . k \tag{17}
\end{equation*}
$$

where $\operatorname{cov}\left(F_{2}^{\Lambda_{1}}, I_{m}^{\Lambda_{\theta}}\right)$ is a covariation between the $F_{2}^{\Lambda_{1}}$ fixed frame of the first image and all the obtained frames of the second ${ }^{I^{l}{ }^{\Lambda_{\theta}}}$, which meet the dependence (14); the 1 variable indexes all the obtained frames of the second image.

After defining the ${ }^{r_{\theta}}$ correlation maximum value

$$
\begin{equation*}
r_{\theta}=\max _{\substack{\zeta \in \Lambda_{\theta} \\ \zeta \in[1 . . k]}}\left(\rho_{\zeta}\right), \quad \theta=2 . . \Theta, \tag{18}
\end{equation*}
$$

between the frames from the set $\Lambda 2$ and $F_{2}^{\Lambda_{1}}$, we define by the $\zeta_{\text {index }}$ the desired frame on each $\{\Lambda \theta\}, \theta$ $=2 . . \Theta$ image and the shift of its coordinates

$$
\begin{equation*}
\Delta_{x}^{\theta}=\left|x_{\text {поч }}^{F_{\zeta}^{\Lambda_{\theta}}}-x_{\text {поч }}^{F_{2}^{\Lambda_{1}}}\right| ; \quad \quad \Delta_{y}^{\theta}=\left|y_{\text {поч }}^{F_{\zeta}^{\Lambda_{\theta}}}-y_{\text {поч }}^{F_{2}^{\Lambda_{1}}}\right| . \tag{19}
\end{equation*}
$$

After shifting all images by the corresponding $\Delta_{x}^{\theta}$ and $\Delta_{y}^{\theta}$ values, we will obtain the set of images of the research object centered with respect to the $F_{2}^{\Lambda_{1}}$ frame of the $\Lambda 1$ image within one pixel.

## 4. Conclusion

| $P_{1}$ | $\ldots$ | $P_{50}$ | $\ldots$ | $P_{100}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | en en c-3 $\frac{1}{3}$ |
| Fig. 1 Entrance set of images |  |  |  |  | Fig. 2 Fixed fragment |

On the basis of the
higher mathematical realization is created.. The entrance set of images is resulted on Fig. 1. On Fig. 2 the fixed fragment $F_{1}^{P_{1}} \quad$ is resulted from Fig. $1 P_{1}$. On Fig. 3 represent result the time of implementation both algorithms (correlation correlation method of images centering; vector flux algorithm of images centering constructed on the basis of combination of total count and correlation analysis for searching fixed fragment on images of set $\left\{P_{n}\right\}, n=2 . . t$.


Fig. 3 Time of implementation searching the fragment in set of images for Correlation and Vector flux methods

The algorithm of images centering constructed on the basis of combination of the total count and correlation analysis methods decrease time of processsing about $5-7 \%$ for one image in comparison with correlatiom method.

That value (interest) increase proportionally to dimension of set. This moment is very impotant, because methods of centering images are one phase of general problem increase resolution of images.

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