Structural-functional modeling for the determination of the company's equilibrium conditions in the dynamic business environment

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The article discusses algorithms for structural and functional modeling, implemented in the form of an optimal control problem in which the motion of systems is described by a first-order differential equation, to determine the optimal period of transition of the enterprise system to the planned equilibrium state and the required values of indicator indicators. The developed model is used to predict the innovative dynamics of the enterprise, the parameters of the model are used with the help of financial and economic indicators of the PJSC "Bashtansky cheese factory". Seven possible equilibrium states have been identified, which can be determined by financial and economic indicators, they summarize a certain level of managerial and technological maturity of the enterprise.

Keywords: *innovative model, structural-functional modeling, equilibrium states, dynamics, system behavior.*

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1. Introduction

To ensure the dynamic development of the enterprise, it is important to define the management of a new type that implements modern formats of possible variants of enterprise development as a socio-economic, which is constantly changing under the influence of factors of the business environment. Given that the company always operates under the influence of endogenous and exogenous factors of the external business environment, the authors believe that there will be no absolute stability and balance for the enterprise. The system loses equilibrium as a result of influence on it, in inconsistency process, uncertainty, deviation from the planned development targets, there are contradictions in the mechanism of action of the system, fluctuations are created and the system fluctuates shown in the papers [1, 2].

If we consider the process of dynamic enterprise development as a transition from one level of stability to another, then we define the criteria for the loss of stability. The matrix of space of equilibrium states of the enterprise allows us to characterize enterprise equilibrium state.

We have identified seven basic levels of enterprise system stability based on work [3, 4].

The problem of optimal control in which the motion of systems is described by a first-order differential equation to determine the optimal period of transition of the enterprise system to the planned equilibrium state on the basis of work [5–7].

2. Description of the model, subject of the model, and research methods

We are invited to apply the mathematical model of optimal rapid transition of the enterprise system to an effective state, which will allow us to formulate scenarios for how to achieve the planned result in a short time by effective methods.

To assess the impact on enterprise dynamics, a set of innovative tools and management activities proposes a model based on a mathematical model of minimum time management based on [8,9].

The general formulation of the problem is this: let the firm's behavior be described by equations:

$$\frac{dy}{dt} = f(t, y, u), \tag{1}$$

where y is unknown function, f is given function, u is control.

The efficiency of the enterprise is evaluated by the functional,

$$J = \int_{t_0}^{T} f_0\left(t, y(t), u(t)\right) dt \to \min \tag{2}$$

among all the trajectories $y(t)$, $t_0 \leqslant t \leqslant T$, which satisfy the following conditions

$$y(t_0) = y_0, \quad y(T) = 0. \tag{3}$$

It should be noted that the initial value of y_0 can be considered as an integral indicator of the state of the enterprise at the time of implementation of management actions.

Thus, under the solution of control problem (1)–(3), there are a pair of functions (y^*, u^*) that give the minimum of the functional J of (2), and for each u of the class. For admissible curations, the function y^* satisfies equation (1) and conditions (3).

The class of valid controls is determined by the constraint $|u| < \infty$.

Such a restriction is natural to account for innovation or investment impacts. Let u be determined by the ratio of the amount of investment invested in the innovative development of the enterprise to the total cost of production. Then, obviously, $|u(t)| \leqslant 1$.

Let us proceed to the description of the proposed model. Suppose that b_1 are resources for production, b_2 is the amount of resources required to produce a unit of production, b_3 is the volume of production that does not require demand, b_4 are the resources expended per unit of production, not in demand. Then the factor of growth of the market value of the enterprise is determined by the formula $m(y) = b_1 - b_2 y$, and the loss factor of the enterprise $n(y) = b_3 + b_4 y$, where y is the output.

Denote by $k(y) = m(y) - n(y) = \alpha(A - y)$, $\alpha = b_2 + b_4$, $A = (b_1 - b_2)/(b_2 + b_4)$. The equation of change of state of the enterprise in the model is:

$$\frac{dy}{dt} = \alpha(A - y)\, y. \tag{4}$$

The general solution of equation (4) is as follows:

$$y(t) = \frac{A\, C\, e^{A\alpha t}}{1 + C\, e^{A\alpha t}}. \tag{5}$$

where C is arbitrary and the parameters α and A are defined above.

By the first of conditions (3) we have $C = \frac{y_0}{A - y_0}$ and

$$y(t) = \frac{A}{1 + (\frac{A}{y_0} - 1)e^{-A\alpha t}}, \quad y_0 \neq 0. \tag{6}$$

Note that the solution (6) is $y(t) > A$, for $t \to \infty$. The functional J looks like:

$$J = \int_{t_0}^{T} \Big(A - \frac{A}{1 + (\frac{A}{y_0} - 1)e^{-A\alpha t}}\Big) dt. \tag{7}$$

We determine the constants A and α from condition (3) and the minimum functional condition J. For the found constants y_0, A, and α by the formula (6), we find the solution $y^*(t)$ and the corresponding optimal control $u^*(t) = k\, y^*(t)$, where $k \in R(0, 1)$.

This functional (7) expresses the criterion of optimality of the enterprise as an economic system, which is expressed in the approximation of the equilibrium state indicators to its limit value. The parameter A provides a characteristic of the most effective development in the given parameters of the enterprise system (this is a dynamic indicator). To optimize the current state to A_{opt}, the current state of the economic system of the enterprise should approach A at the expense of the parameters that can be managed. These parameters, which are objects of administrative actions, include parameters which determine the innovative activity of the enterprise and the main indicators determining the enterprise as a social and economic system.

The presence of additional parameters A, α, which are related to the main control indices, can be interpreted as a bifurcation field. The economic model studies generally use a discrete time, which is related to the reporting period. The model is described by equation (4) with conditions (3) and functional (7). Therefore, the construction of the model is reduced to determining its parameters A, α, y_0.

The indicators for constructing a model of the field of bifurcation are given in Table 1.

Table 1. Main management indicators.

No	Conformity	Indicator
1.	*Indicators of the financial and economic group*	
	$b_1 \to E_{21}$	Indicator of total liquidity
	$b_2 \to E_{14}$	Indicator of maneuverability of equity
	$b_3 \to E_{11}$	Indicator of financial stability
	$b_4 \to E_{42}$	Indicator of asset profitability ratio (ROA)
2.	*Indicators of the production group*	
	$b_1 \to P_{11}$	The coefficient of efficiency of use of the basic production assets of the enterprise
	$b_2 \to P_{14}$	Indicator of the share of production working capital in working assets
	$b_3 \to P_{16}$	Indicator of fixed asset renewal
	$b_4 \to P_{22}$	Indicator of loading of production facilities

The methodological composition of the economic-mathematical modeling is based on the data of each PJSC "Bashtany cheese factory" [10].

Table 2. The main guiding indicators for PJSC "Bashtany cheese factory" for the period of 2008–2016.

Indicators	2008	2009	2010	2011	2012	2013	2014	2015	2016
E_{11}	0.834	0.732	0.568	0.616	0.514	0.587	0.666	0.674	0.532
E_{14}	0.730	1.118	1.141	1.637	1.348	1.083	1.680	1.702	2.117
E_{31}	0.390	0.325	0.311	0.317	0.442	0.421	0.298	0.282	0.423
E_{42}	0.120	0.121	0.092	0.000	0.051	0.065	0.100	0.000	0.133
E_{21}	3.248	2.459	1.491	1.927	1.409	1.524	2.350	2.485	1.776
P_{11}	4.356	4.474	6.569	5.570	4.843	4.619	4.547	4.067	4.868
P_{14}	0.355	0.192	0.256	0.268	0.256	0.260	0.217	0.201	0.219
P_{15}	1.174	1.939	2.102	3.066	2.179	1.700	3.627	4.277	4.924
P_{16}	1.648	1.188	1.057	1.241	1.146	1.146	1.021	1.009	1.030
P_{22}	0.134	0.088	0.050	0.253	0.165	0.125	0.079	0.093	0.179

The group of production and financial indicators is calculated as the main indicators of determining the state of equilibrium of the enterprise and the possibility of its transition to the optimal state, that is, the possibility of transition of the enterprise to the state in which management actions will have the greatest effect, for the development of the enterprise, such state we are defined as a zone of managed attractor.

For each enterprise indicator of optimal transition are defined. The choice of metrics by criterion is small errors and great ones of authenticity.

1. The first stage of the calculation of the model, the calculation of the indicator A,

$$
\begin{aligned}
&A_{2008} = \tfrac{3.248-0.834}{0.73+0.12} = 2.84,\\
&A_{2009} = \tfrac{2.459-0.732}{0.73+0.121} = 2.029,\\
&A_{2010} = \tfrac{1.491-0.568}{1.141+0.092} = 0.7486,\\
&A_{2011} = \tfrac{1.927-0.616}{1.637+0.000} = 0.8009,\\
&A_{2012} = \tfrac{1.409-0.514}{1.348+0.051} = 0.6397,\\
&A_{2013} = \tfrac{1.524-0.587}{1.083+0.065} = 0.8162,\\
&A_{2014} = \tfrac{2.350-0.666}{1.680+0.100} = 1.0024,\\
&A_{2015} = \tfrac{2.485-0.674}{1.702+0.0} = 1.0640,\\
&A_{2016} = \tfrac{1.776-0.532}{2.117+0.1330} = 0.5529.
\end{aligned}
$$

2. The second stage of model calculation, calculation α,

$$
\begin{aligned}
&\alpha = (b_2 + b_4),\\
&\alpha_{2008} = 0.73 + 0.12 = 0.85,\\
&\alpha_{2009} = 0.73 + 0.121 = 0.851,\\
&\alpha_{2010} = 1.141 + 0.092 = 1.233,\\
&\alpha_{2011} = 1.637 + 0.0 = 1.637,\\
&\alpha_{2012} = 1.348 + 0.051 = 1.399,\\
&\alpha_{2013} = 1.083 + 0.065 = 1.148,\\
&\alpha_{2014} = 1.680 + 0.10 = 1.78,\\
&\alpha_{2015} = 1.702 + 0.0 = 1.702,\\
&\alpha_{2016} = 2.117 + 0.133 = 2.25.
\end{aligned}
$$

3. Calculation y_0 is carried out by the formula previously derived

$$
\begin{aligned}
&Y_0 = \tfrac{A}{1+(A-1)e^{-A\alpha}},\\
&Y_{(2008)} = \tfrac{2.84}{1+(2.84-1)e^{-2.84\cdot 0,85}} = 2.4386,\\
&Y_{(2009)} = \tfrac{2.029}{1+(2.029-1)e^{-2.099\cdot 0.851}} = 1.7151,\\
&Y_{(2010)} = \tfrac{0.7486}{1+(0.7486-1)e^{-0.7486\cdot 1.233}} = 0.8317,\\
&Y_{(2011)} = \tfrac{0.8009}{1+(0.8009-1)e^{-(0.8009\cdot 1.637}} = 0.8463,\\
&Y_{(2012)} = \tfrac{0.6397}{1+(0.6397-1)e^{-(0.6397\cdot 1.399}} = 0.7501,\\
&Y_{(2013)} = \tfrac{0.8162}{1+(0.8162-1)e^{-(0.8162\cdot 1.148)}} = 0.8795,\\
&Y_{(2014)} = \tfrac{1.029}{1+(1.029-1)e^{-(1.029\cdot 1.78)}} = 1.0242,\\
&Y_{(2015)} = \tfrac{1.064}{1+(1.064-1)e^{-(1.064\cdot 1.702)}} = 1.0529,\\
&Y_{(2016)} = \tfrac{0.5529}{1+(0.5529-1)e^{-(0.5529\cdot 2.25)}} = 0.6347.
\end{aligned}
$$

4. To calculate the equilibrium state corridor, determine the portrait of the cheese factory. For this we will select indicators of equilibrium states of the enterprise A.

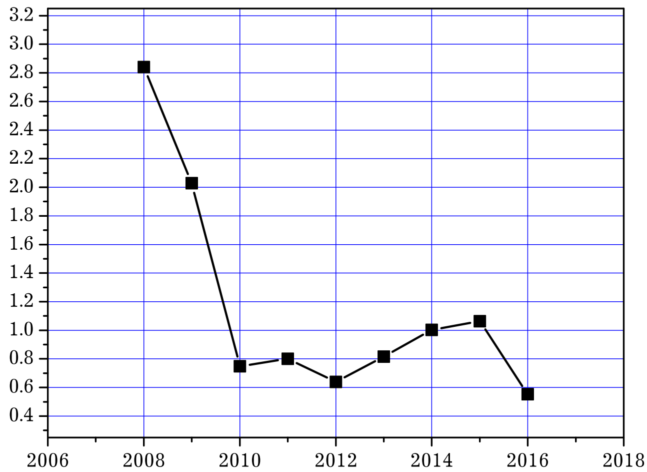

Fig. 1. Schedule of change of parameter A (equilibrium state) for PJSC "Bashtany cheese factory".

5. Calculation of the average value of $M(A)$ state of equilibrium of the enterprise and the phase of the cycle in which the enterprise is located.

$$\begin{aligned} M(A) &= \frac{\sum A}{9} \\ &= \frac{2.84 + 2.029 + 0.7486 + 0.8009 + 0.6397 + 0.8162 + 1.0024 + 1.064 + 0.5529}{9} = 1.1660. \end{aligned}$$

6. Structural-functional modeling of minimization of the determined functional of the system's behavior and possible equilibrium states.

6.1. Determining the optimal transition to the most effective state of the enterprise development by the given parameters for a certain period of time. This state is determined with the help of

$$u^*(t) = k\,y^*(t),$$

$y^*(t)$ is innovative activity of the system.

6.2. The justification of the choice of a tunnel transition in the most effective state of the enterprise as an economic system, is carried out with the help of a functional by the choice of the period of implementation of the transition. To do this, use the value of the functional.

$$\int_{t_0}^{T} \frac{dt}{1+\beta e^{-\nu t}} = \left[t + \frac{1}{\nu}\ln(1+\beta e^{-\nu t}) \right] \Bigg|_{t_0}^{T}. \tag{8}$$

In the formula (8) $\nu = \alpha A$, $\beta = A/y_0 - 1$.

6.3. We will calculate the term of the possible transition of the system by the defined parameters of functioning in the most optimal state. The term is determined depending on the main parameter of innovative capabilities - innovation activity.

Structural-functional modeling of the possible transition period of the economic and social system of the enterprise to the optimal set state of equilibrium, that is, the definition and formation of the pool attractor PJSC "Bashtany cheese factory".

6.4. Consider the function $y^*(t)$, the base period $t_0 = 2016$.

The measurement effect from the introduction of innovative management technologies will be achieved within the period from 1 to 15 years. That is why the dimension of the temporal parameter is given $T = 1.5$ years. To understand how effective is the innovative technology that we have implemented, it will take time. Usually, companies see it only after 7–9 months. We took a period of 1.5 years to understand how long the enterprise would need to go into a dynamic development mode if it remains in the same mode of innovation movement,

$$\begin{aligned}
&\nu = +A\alpha = 1.166\alpha,\\
&\beta = \tfrac{A}{y_0} - 1 = \tfrac{1.166}{0.6347} - 1 = 0.8371,\\
&\nu = 1.166 \cdot 2.25 = 2.6235.
\end{aligned}$$

6.5. Calculate the coefficient of innovation activity $y^*(t)$:

$$\begin{aligned}
y^*(t) &= 1.5 + \frac{1}{2.6235}\ln(1 + 0.8371e^{-2.6235\cdot 1.5}) - \frac{1}{2.6235}\ln(1 + 0.8371) = 1.2752,\\
y^* &= 1.2752;\\
k &= 0.5,\\
u^*(t) &= 0.5 \cdot 1.2752 = 0.6376.
\end{aligned}$$

This value indicates that for the transition and preservation of the dynamic development of the enterprise it is necessary to form innovative activity at the level not less than 63.76% in the total volume, in order to transfer in 1.5 years to the system of the enterprise in the planned state of development.

To monitor the state of the system and to prevent adverse effects in a timely manner, it is necessary to constantly analyze the state of the enterprise. Method of assessing the level of financial and economic and social stability of the enterprise on the basis of qualitative and quantitative analysis of economic, production, personnel, informatization and innovative processes, which allows us to form analytical forecasts of the enterprise development in the current conditions of the business environment [2, 3, 9].

Table 3. Limit thresholds for the functional state of the enterprise.

Interval thresholds	Functional characteristic of system stability
$0.4 \leqslant k_i \leqslant 0.5$	Normal equilibrium (small fluctuations of indicators, the possibility of development)
$0.5 \leqslant k_i \leqslant 0.7$	Relative equilibrium (more amplitude of fluctuations of indicators, capacity for development persists)
$0.7 \leqslant k_i \leqslant 0.8$	Relative imbalance (increase in oscillation amplitude, quasi-development)
$0.8 \leqslant k_i \leqslant 0.95$	Critical imbalance (the crisis state of the development of the occurrence of the field of bifurcation)
$0.7 \leqslant k_i \leqslant 0.8$	Limit imbalance (the fluctuation of indicators fades out and the enterprise in the pool of attractors determines its vector of development)
$0.6 \leqslant k_i \leqslant 0.7$	Limit equilibrium (small fluctuations of indicators induce the enterprise to develop dynamically according to the given vector)
$0.4 \leqslant k_i \leqslant 0.5$	Optimal equilibrium (slight fluctuations of parameters, optimal ability of dynamic development

Systematicity and complexity of approaches requires analyzing the enterprise system from an extrapolation-cyclical position, that is, on the basis of the analysis of the real state of the enterprise, to form models of state space and to determine opportunities and inclination for dynamic development.

From the point of view of the theory of innovative dynamics, the socio-economic system of an enterprise may not always be stable if it evolves. It is prone to transformational actions of external and internal factors. Whenever an enterprise system approaches the critical values of external parameters, sudden unforeseen structural changes and chaos occur in it. To counteract such processes, a certain stabilizer/inhibitor must be introduced into the system. The authors consider such a stabilizer or inhibitor of the system the use of control technology in their transformed form, namely: innovation as a strategic aspect of dynamic development

3. Conclusions

On the basis of the performed structural and functional modeling of the implementation, it is proposed to use the mathematical model of the optimal rapid transition of the enterprise system to an effective state, which will allow the formation of scenarios, how to achieve the planned result in a short time by effective methods.

The systems approach requires to analyze the system of the enterprise with an extrapolation-cyclic position, that is, based on the analysis of the real state of the enterprise to form a model of the space of states and to identify the opportunities and the tendency to dynamic development.

When forecasting changes in enterprise parameters, you can identify the prevailing trends in the vector of enterprises motion, possibilities of influence on separate elements of the system or in general on the whole system, and possibilities of regulating fluctuations in the system and creating contour conditions for controlled attractors to ensure the dynamic development of the enterprise.

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Структурно-функціональне моделювання для визначення стану рівноваги підприємства в умовах динамічного бізнес-середовища

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У статті розглядаються алгоритми структурно-функціонального моделювання, реалізовані у вигляді задачі оптимального керування, в якій рух систем описується диференціальним рівнянням першого порядку, для визначення оптимального періоду переходу системи підприємства до планового стану рівноваги та необхідних значень індикаторних показників. Розроблена модель використовується для прогнозування інноваційної динаміки підприємства, параметри моделі використовуються за допомогою фінансово-економічних показників заводу ПАТ "Баштанський сирзавод". Виявлено сім можливих станів рівноваги, які можуть бути визначені за допомогою фінансово-економічних показників, вони узагальнюють певний рівень управлінської та технологічної зрілості підприємства.

Ключові слова: *структурно-функціональне моделювання, стан рівноваги, динаміка, поведінка системи.*