Accelerating the Parametric Identification of the Monod System Using Explicit and Implicit Schemas

Roman Pasichnyk, Yuriy Pigovsky, Orysia Gunderych

Abstract – **current study is a research of different schemas identifying the Monod nonlinear stiff ODE. The schemas provide better performance and accuracy in comparison with the classic implicit ODE solvers.**

Keywords – **parametric identification, Monod system, stiff ODE, explicit schema, implicit schema.**

I. INTRODUCTION

The Monod system is widely used as a model of fermenting processes. Refining the speed and accuracy of parametric identification of such a model is of much interest in chemical and technological research.

Current study aims to discover the abilities to accelerate the identification speed using a limited explicit schema and compares these results with those obtained using the explicit-implicit schema of [1].

II. STIFFNESS OF THE MONOD MODEL

Isothermal fermentation processes can be modeled using a Cauchy problem for the autonomous Monod ODE system. The numeric approximation of the problem solution can be obtained using the Euler explicit schema

$$X_{k+1} = (1 + h\, l_{1,k}) X_k, \qquad S_{k+1} = (1 + h\, l_{2,k}) S_k, \tag{1}$$

$$P_{k+1} = P_k + h A_7 A_3 \frac{S_k X_k}{A_4 + S_k}\left(1 - \frac{2P_k}{S_0}\right), \tag{2}$$

with initial conditions $X_0 > 0,\ S_0 > 0,\ P_0 = 0,$ where X_k is the biomass concentration at k^{th} time discrete, S_k is the substrate concentration, P_k the is product (ethanol) concentration, $l_{1,k}$ and $l_{2,k}$ are computed as

$$l_{1,k} = A_1 \frac{S_k}{A_5 + P_k} - A_2, \qquad l_{2,k} = -\frac{A_3 X_k}{A_4 + S_k} \tag{3}$$

Equations (1) are conditionally stable and don't loose their physical sense ($X_k \ge 0,\ S_k \ge 0,\ k = 0,1,2\ldots$) when the conditions

$$-1 \le h\, l_{1,k}, \quad -1 \le h\, l_{2,k} \tag{4}$$

are satisfied. Eq. (2) is always stable, because its right side is always greater than zero.

The large absolute value of $l_{1,k}$ or $l_{2,k}$ require reduced value of step h in order to satisfy (4). Small values of h leads to increased computation time in schema (1)–(3), because every next k+1th time discrete is increased by $h = t_{k+1} - t_k$. Moreover, in the case when one l is drastically greater than the opposite one, i.e. $|l_{1,k}| >>> |l_{2,k}|$ or $|l_{2,k}| >>> |l_{1,k}|$, the product $h\, l_{2,k}$ or $h\, l_{1,k}$ can lead to a machine zero, refusing the computation accuracy. Such a phenomena is known as the stiffness of ODE system.

The largest stiffness is observed at the end of the process, when $X_k \to 0$, $S_k \to 0$ and $l_{1,k} \to -A_2$, $l_{2,k} \to 0$ leading to $|l_{1,k}| >>> |l_{2,k}|$.

III. THE EXPLICIT-IMPLICIT AND LIMITED SCHEMAS

In order to attack this problem, a new explicit-implicit method is developed.

Theoretical research, conducted in [1] leads to the following equation for boundary substrate value

$$S_{bound} = -h^* S'_{real,m}, \tag{5}$$

where $S'_{real,m}$ is a finite difference approximation of substrate derivate at time *m*, when a half of substrate was consumed, i.e. $S_{real,m} \approx 0.5 S_0$; h^* is maximal step value.

The eq. (5) defines time area, where an explicit schema is still applicable, i.e. all k=0,1,2… for which $S_k > S_{bound}$.

The limited schema is based on the observation that $S_k \to 0$ and $P_k \to 0.5 S_0$ when $k \to \infty$. So the model (1)–(2) can be reduced to

$$X_k = X_q \exp[-A_2 (t_k - t_q)], \ S_k = 0, \ P_k = P_q, \tag{6}$$

starting from some q^{th} time node ($k = q, q+1, \ldots$), where $S_q \le S_{\lim}$. System is solved using the explicit (1)-(2) schema for $0 \le k \le q$ and using (6) for the rest $k > q$.

IV. RESULTS AND CONCLUSION

The explicit-implicit scheme (5) reduces the duration of identification procedure up to 46%, and gets the identification error of 32% in relation to the classic implicit scheme [1].

The new result is the ability to reduce the duration of identification up to 29% using the limited schema (6) without enormous increases in identification errors, i.e. no more than 5% in relation to X_k, S_k and P_k maximal values.

REFERENCES

[1] R. Pasichnyk, Y. Pigovsky, "Identification of stiff nonlinear models of chemical-engineering systems," *Scientific and technical journal of the Institute for Information Recording of Ukraine NAS "Registration, storage and data processing,"* vol. 12, No. 1, pp. 12-22, 2010 (in Ukrainian).

Roman Pasichnyk, Yuriy Pigovsky, Orysia Gunderych - Ternopil National Economic University, 9 Yunosti Str., Ternopil, 46000, UKRAINE, E-mail: pigovsky@gmail.com