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# LOGARITHMIC DERIVATIVE ESTIMATES FOR SUBHARMONIC FUNCTIONS

For a measurable set $E \subset [0,\infty)$, the upper and lower densities are defined as

$$\overline{D}(E) = \limsup_{r\to\infty} \frac{m(E\cap[0,r])}{r}, \quad \underline{D}(E) = \liminf_{r\to\infty} \frac{m(E\cap[0,r])}{r},$$

where $m(E)$ there is Lebesgue measure of a set $E$.

For any subharmonic function $u$ we have the following Poisson-Jensen formula in the disk $C(0,R) = \{\zeta : |\zeta| < R\}$

$$u(z) = \frac{1}{2\pi}\int_0^{2\pi} u\left(\mathrm{Re}^{i\varphi}\right)\mathrm{Re}\left(\frac{\mathrm{Re}^{i\varphi}+z}{\mathrm{Re}^{i\varphi}-z}\right)d\varphi + \int_{C(0,R)} \ln\left|\frac{R(z-\zeta)}{R^2 - z\overline{\zeta}}\right| d\mu(\zeta),$$

where $\mu = \mu_u$ there is Riesz measure of the function $u$. Let us consider the function

$$q(z) = \frac{1}{2\pi}\int_0^{2\pi} u\left(\mathrm{Re}^{i\varphi}\right)\left(\frac{d}{dz}\right)\left(\frac{\mathrm{Re}^{i\varphi}+z}{\mathrm{Re}^{i\varphi}-z}\right)d\varphi + \int_{C(0,R)} \left(\frac{d}{dz}\right)\ln\frac{R(z-\zeta)}{R^2 - z\overline{\zeta}} d\mu(\zeta).$$

We denote $w(z) = zq(z)$.

**Theorem 1.** Suppose that $u$ is subharmonic function in $\mathbb{C}$, and $u(0) \neq -\infty$. If $1 < r < R$, we have

$$\frac{1}{2\pi}\int_0^{2\pi} \left|\mathrm{Re}\, w\left(re^{i\theta}\right)\right| d\theta \le \frac{2T(r,u) - u(0)}{\log\frac{R}{r}}.$$

**Theorem 2.** Let $0 < \delta < 1$ and $k > 1$. Let $u$ be a subharmonic function in $\mathbb{C}$, $u(0) = 0$ and harmonic in a neighborhood of the origin. Then there exist a measurable set $E \subset [0,\infty)$ with $\overline{D}(E) < \delta$ and a constant $C = C(\delta) > 0$, such that

$$\int_0^{2\pi} \left|q\left(re^{i\theta}\right)\right| d\theta \le C\frac{T(kr,u)}{r}, \quad r \notin E.$$

Above mentioned results are joint work with I. E. Chyzhykov.