

# ОПРАЦЮВАННЯ ТА ПЕРЕТВОРЕННЯ ВИМІРЮВАЛЬНИХ СИГНАЛІВ

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## DYNAMIC PROPERTIES OF TRANSDUCERS TESTING USING WHITE NOISE EXCITATION. PART 2: FREQUENCY DOMAIN

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*This article deals with experimental non-parametric methods of parameters qualifying dynamic properties of transducers of voltage/voltage type using external stochastic input function (white noise) and single-sided power spectral density functions. To compute the power spectral density functions Brüel & Kjør 3550 digital signal analyser has been applied.*

### Determination of transducers dynamics parameters on the grounds of the course of power spectral density functions

The single-sided power spectral density (PSD)  $G_y(\omega)$  of the output object signal  $y(t)$  is connected with the module of spectral transfer function of examined system  $|K(j\omega)|$  by the dependence [1-7, 9-14]:

$$G_y(\omega) = |K(j\omega)|^2 G_x(\omega), \quad (1)$$

where  $G_x(\omega)$  is PSD of  $x(t)$  input signal.

For the white noise characterised by constant PSD

$$G_x(\omega) = a = \text{const}$$

the dependence (1) takes the form of:

$$G_y(\omega) = |K(j\omega)|^2 a, \quad (2)$$

where after some modifications one gets:

$$|K(j\omega)| = K(\omega) = [G_y(\omega) / a]^{(1/2)} \quad (3)$$

Defining PSD in decibels on the basis of the equation (2) one gets:

$$\begin{aligned} G_y(\omega)_{dB} &= 10 \log G_y(\omega) = \\ &= 10 \log |K(j\omega)|^2 + 10 \log a \end{aligned} \quad (4)$$

The dependence (4) makes it easy to compare PSD with the amplitude-frequency logarithmic characteristics of system  $L(\omega)_{dB}$ :

$$\begin{aligned} L(\omega)_{dB} &= 20 \log |K(j\omega)| = \\ &= 10 \log |K(j\omega)|^2 \end{aligned} \quad (5)$$

### 1.1. Inertial transducer of the 1<sup>st</sup> order

Comparing dependence (4) and (5) it is easy to notice that  $G_y(\omega)_{dB}$  and  $L(\omega)_{dB}$  characteristics differ only in  $10 \log a$  constant value. Taking this fact into consideration the refraction pulsation  $\omega_z$  can be calculated from the course of PSD, in the same way as in the case of the amplitude-frequency logarithmic characteristic [11], approximating  $G_y(\omega)_{dB}$  with two half-lines characterised by equations (for  $K(\omega) = k / (1 + \omega^2 T^2)^{(1/2)}$ ), where:  $k$  - gain coefficient,  $T$  - time-constant):

$$G_y(\omega)_{dB} = \begin{cases} 20 \log k + 10 \log a = \\ G_{y0} & \omega \leq 1/T \\ 20 \log k + 10 \log a + \\ -20 \log \omega T & \omega \geq 1/T \end{cases} \quad (6)$$

According to dependence (6) a slope of a half-line for  $\omega > 1/T$  equals  $-20$  dB/decade. The downgrade of the characteristic:  $\Delta G_y(\omega)_{dB} = G_y(\omega)_{dB} - G_{y0}$  for  $\omega = 1/T$  equals  $-3$  dB. On the basis of the refraction pulsation time-constant of transducer can be defined:

$$T = 1 / \omega_z. \quad (7)$$

Figure 1 shows an exemplary course of PSD  $G_y(\omega)_{dB}$ , defined for the model of the 1<sup>st</sup> order transducer, using a rectangular data window and  $N = 1000$  averaging number.

In table 1 there is the comparison of time-constant achieved as an outcome of examining of two transducer

models of the 1<sup>st</sup> order applying step input function and white noise. In the case of step function the time-constant is obtained by the course of step response [8].

Table 1

**Time-constant comparison of two transducers models of the 1<sup>st</sup> order**

Parameter	Transducer No	Step excitation	Noise excitation
T [μs]	1	480	452
	2	102	103

**1.2. Oscillatory transducer of the 2<sup>nd</sup> order**

There are two parameters defining the dynamics of the 2<sup>nd</sup> order oscillatory transducer and they are: the damping ratio  $\xi$  and the undamped natural pulsation  $\omega_0$ . The first of these values is calculated on the basis of the

analysis of the course of the amplitude-frequency characteristic of the system from dependence [5]:

$$K(\omega_r)/K_0 = 1/2\xi(1 - \xi^2)^{(1/2)} \Rightarrow \xi, \quad (8)$$

where:  $K(\omega_r)$ ,  $\omega_r$  – the resonance peak coordinates,  $K_0$  – the amplitude of the initial characteristic  $K(\omega)$ .

Taking into account equation (3), dependence (8) can take the form of:

$$[G_y(\omega_r)/G_{y0}]^{(1/2)} = 1/2\xi(1 - \xi^2)^{(1/2)}, \quad (9)$$

where after modifications one gets:

$$\xi_{1,2} = \left\{ \left[ 1 \pm \left[ 1 - \left( \frac{G_{y0}}{G_y(\omega_r)} \right) \right]^{(1/2)} \right] / 2 \right\}^{(1/2)} \quad (10)$$

An exemplary course of PSD  $G_y(\omega)$  function, determined for the model of the 2<sup>nd</sup> order oscillatory transducer is shown in figure 2.

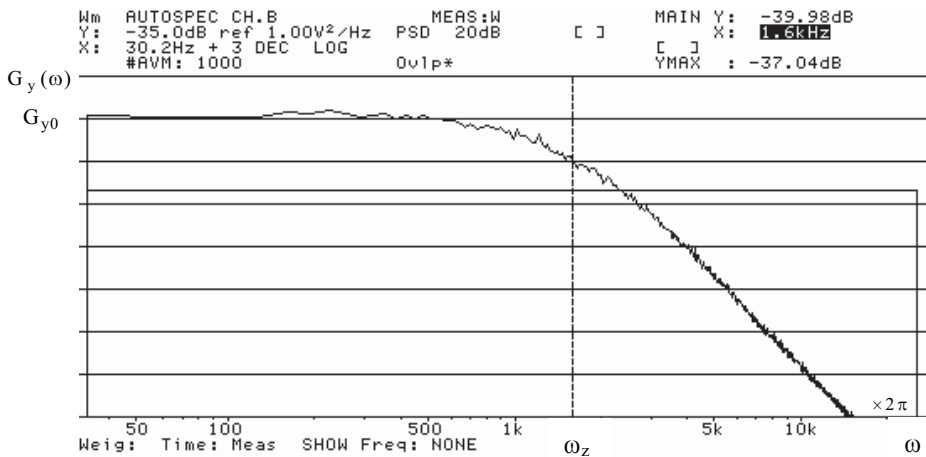


Fig. 1. Power spectral density  $G_y(\omega)$  dB for the 1<sup>st</sup> order transducer model

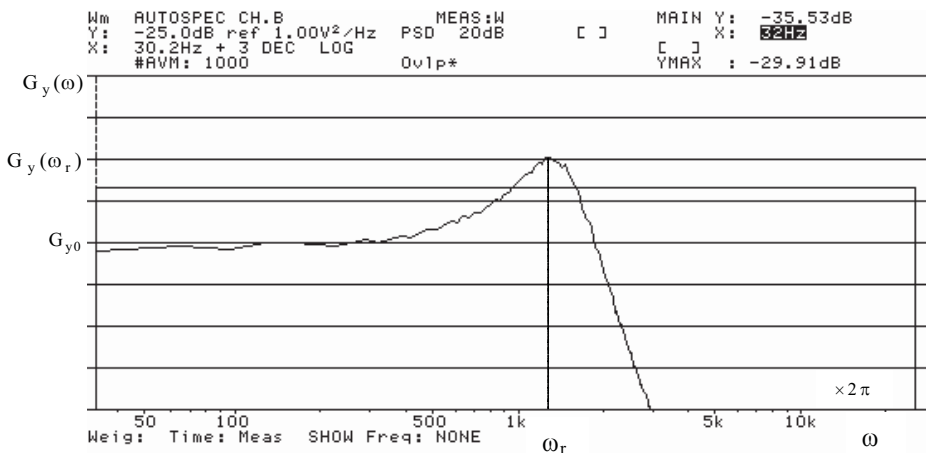


Fig. 2. Power spectral density  $G_y(\omega)$  dB of the 2<sup>nd</sup> order oscillatory transducer

The natural pulsation  $\omega_0$  can be determined on the basis of the resonance pulsation  $\omega_r$  and the damping ratio  $\xi$  from dependence:

$$\omega_0 = \omega_r / (1 - 2\xi^2)^{(1/2)}. \quad (11)$$

From equations (10) and (11) one gets a couple of solutions.

Comparison of damping ratio and natural pulsation of two 2<sup>nd</sup> order transducer models obtained by using step and noise excitation is shown in table 2. In the case of noise input function  $\xi$  and  $\omega_0$  are calculated on the basis on equation (10) and (11).

Table 2

**Comparison of parameters  $\xi$  and  $\omega_0$  for two transducers models of the 2<sup>nd</sup> order**

Parameter	Transducer No	Step excitation	Noise excitation
$\xi$ [-]	1	0,539	0,528
	2	0,294	0,296
$\omega_0$ [rad/s]	1	8778	8339
	2	9005	8840

**Determination of transducers dynamics parameters on the basis of the course of cross-spectral density functions**

The single-sided cross-spectral density function (CSD)  $G_{xy}(\omega)$  of the input  $x(t)$  and output  $y(t)$  signals of transducer is connected with the spectral transmittance of the examined system  $K(j\omega) = K(\omega)\exp[-j\varphi(\omega)]$  by the dependence:

$$G_{xy}(j\omega) = K(j\omega)G_x(\omega), \quad (12)$$

where  $G_x(\omega)$  is CSD of the  $x(t)$  input signal.

Expression (12) can be written as:

$$\begin{aligned} G_{xy}(\omega) \exp[-j\Phi_{xy}(\omega)] &= \\ &= K(\omega) \exp[-j\varphi(\omega)] G_x(\omega) \end{aligned}, \quad (13)$$

where:

$$G_{xy}(\omega) = |G_{xy}(j\omega)| = |K(j\omega)| G_x(\omega), \quad (14)$$

$$\begin{aligned} \Phi_{xy}(\omega) &= \arg\{G_{xy}(j\omega)\} = \\ &= \arg\{K(j\omega)\} = \varphi(\omega) \end{aligned}. \quad (15)$$

In the case of the input signal in the form of white noise with constant PSD

$$G_x(\omega) = a = const$$

dependence (14) takes the form of:

$$G_{xy}(\omega) = |K(j\omega)| a, \quad (16)$$

and further:

$$K(\omega) = |K(j\omega)| = G_{xy}(\omega) / a. \quad (17)$$

Defining CSD in dB, on the basis of equation (16) one gets:

$$\begin{aligned} G_{xy}(\omega)_{dB} &= 10 \log G_{xy}(\omega) = \\ &= 10 \log |K(j\omega)| + 10 \log a \end{aligned}. \quad (18)$$

**2.1. Inertial transducer of the 1<sup>st</sup> order**

Taking into consideration (18), (5) and the analyses described in 1.1 one can assume that in the case of CSD refraction pulsation  $\omega_z$  can be graphically defined after  $G_{xy}(\omega)_{dB}$  course approximation with two half-lines expressed by equations:

$$G_{xy}(\omega)_{dB} = \begin{cases} 10 \log k + 10 \log a = \\ = G_{xy0} & \omega \leq 1/T \\ 10 \log k + 10 \log a + \\ -10 \log \omega T & \omega \geq 1/T \end{cases} \quad (19)$$

A slope of a half-line for  $\omega > 1/T$  equals  $-10$  dB/decade and characteristic downgrade  $\Delta G_{xy}(\omega)_{dB} = G_{xy}(\omega)_{dB} - G_{xy0}$  for  $\omega = 1/T$  equals  $-1,5$  dB. The time-constant of a transducer is calculated from dependence (6).

An additional information is included in phase characteristic CSD (15), the course of which equals to phase-frequency transducer characteristic.

In figure 3 there are some exemplary courses of  $G_{xy}(\omega)_{dB}$  module and  $\Phi_{xy}(\omega)$  phase characteristics, calculated for the 1<sup>st</sup> order transducer module.

In table 3 there is the comparison of time-constant achieved for two transducer models of the 1<sup>st</sup> order applying step input function and white noise and cross-spectral density function.

Table 3

**Time-constant comparison of two transducers models of the 1<sup>st</sup> order**

Parameter	Transducer No	Step excitation	Noise excitation
T [μs]	1	480	474
	2	102	100,5

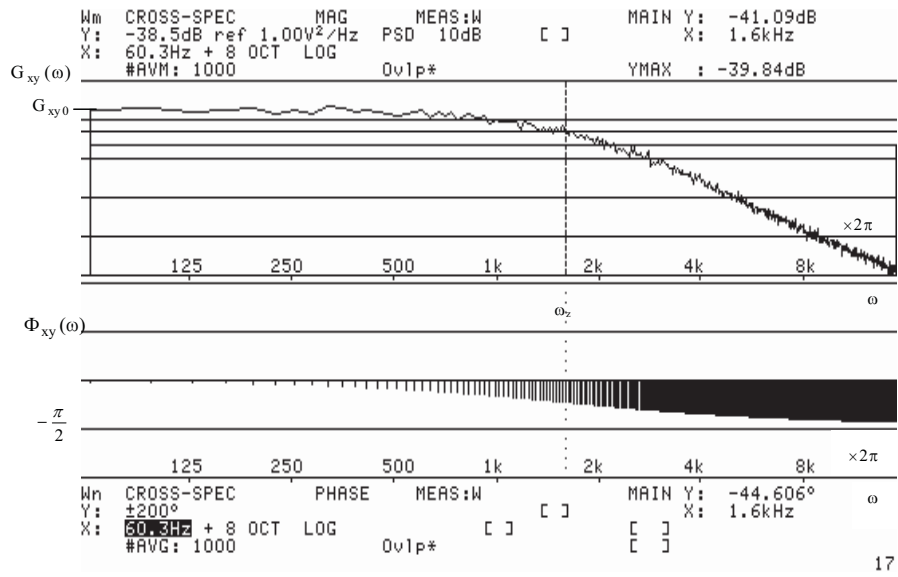


Fig. 3.  $G_{xy}(\omega)$ dB and  $\Phi_{xy}(\omega)$  characteristics defined for the 1<sup>st</sup> order transducer model

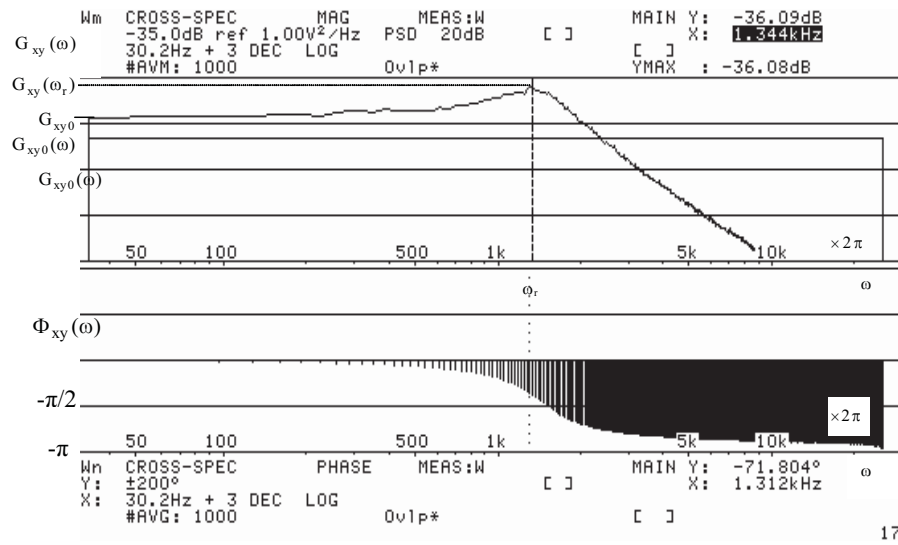


Fig. 4.  $G_{xy}(\omega)$  and  $\Phi_{xy}(\omega)$  characteristics defined for the 2<sup>nd</sup> order oscillatory transducer model

**2.2. Oscillatory transducer of the 2<sup>nd</sup> order**

Exemplary courses of module  $G_{xy}(\omega)$  and phase  $\Phi_{xy}(\omega)$  CSD characteristics, defined for the 2<sup>nd</sup> order transducer model are shown in figure 4.

Regarding equation (8) and (16) one gets the dependence, out of which damping ratio  $\xi$  is calculated:

$$G_{xy}(\omega_r)/G_{xy0} = 1/2\xi(1-\xi^2)^{(1/2)} \Rightarrow \xi, \quad (20)$$

where after some modifications one gets:

$$\xi_{1,2} = \left\{ \left[ 1 \pm \left[ 1 - \left( \frac{G_{xy0}}{G_{xy}(\omega_r)} \right)^2 \right]^{(1/2)} \right] \right\}^{(1/2)} \quad (21)$$

The natural pulsation  $\omega_0$  is calculated from equation (11).

Comparison of damping ratio and natural pulsation of two 2<sup>nd</sup> order transducer models obtained by using step

and noise excitation is shown in table 4. In the case of noise input function  $\xi$  and  $\omega_0$  are calculated on the basis on equation (21) and (11).

Table 4

**Comparison of parameters  $\xi$  and  $\omega_0$  for two transducers models of the 2<sup>nd</sup> order**

Parameter	Transducer No	Step excitation	Noise excitation
$\xi$ [-]	1	0,539	0,506
	2	0,294	0,307
$\omega_0$ [rad/s]	1	8778	8459
	2	9005	8661

### Conclusion

The methods of defining dynamic properties of transducers using noise as testing signals have many advantages and these are [3, 5]:

- the possibility of using random noise, which occurs during normal work of the system;
- the possibility of defining dynamic parameters in the presence of noise (the possibility of eliminating object's own noise influence, under the condition that the noise is noncorrelated with the input signal);
- the measurement time is unlimited as it happens for example in the case of step function.

That is why the application of probabilistic methods is deliberate when:

- useful signals of transducers are the stochastic signals;
- work conditions of transducers do not allow to apply determined testing signals;
- there is no possibility of isolating the examined transducer from noise, as an outcome of which there are huge adulterations of transducer responses to determined testing function;
- getting the determined testing signal is difficult because of the requirements concerning its shape.

As to the disadvantages of the mentioned methods, there is the necessity of determining and analysing statistic characteristics of random signals (in the discussed cases cross-correlation and power spectral density functions). This is, however, more complicated and requires much more time and technical agents than in the case of applying determined signals.

The results shown in tables 1-4 (and tables 1-2 in part I) show basically a bit greater scatter of parameter values in the case of noise excitation. Taking into account a big variance of correlation and spectral function estimators numerically calculated using FFT, it is necessary to use averaging which increases the measurement time.

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