On the New Hyperbolic Function Solutions to the (2+1)-Dimensional BKK System

Hasan Bulut[1*], Tukur Abdulkadir Sulaiman[1]

[1]Firat University, Faculty of Science, Department of Mathematics, Elazig, TURKEY,
E-mail: hbulut@firat.edu.tr , mtukur74@yahoo.com

***Abstract* – In this paper, we successfully implement the powerful sine-Gordon expansion method the (2+1)-dimensional BKK system. We succeed in constructing some new analytical hyperbolic function solutions. We check all the analytical solutions by using Wolfram Mathematica 9, and they are indeed verified to be the solutions of (2+1)-dimensional BKK system. We also plot the two- and three-dimensional surfaces for all the analytical solutions obtained in this paper using the same computer program. We finally, submit a comprehensive conclusion.**

Key words – The sine-Gordon expansion method; the (2+1)-dimensional BKK system; hyperbolic function solution

I. Introduction

In recent years, some physical problems have been modelled by using mathematical structures. Then, many powerfull method have been applied to them by finding new properties of them. It has becomes very important practice to investigate the solution behaviors to such nonlinear mathematical models due to the great impact they have in our life daily activities. These equations describe models that arise in the fields of science, mathematical physics and engineering, such as Biology, plasma physics, fluid mechanics etc. Various approaches [1-6] by diferent researchers have been implemented to investigate the solutions to such class of equations.

In this study, we implement the powerful sine-Gordon expansion method (SGEM) to the (2+1)-dimensional Broer-Kaup-Kupershmidt (BKK) system [7] given as follows:

$$\begin{aligned} &u_{ty}-u_{xxy}+2\left(uu_x\right)_y+2v_{xx}=0,\\ &v_t+v_{xx}+2\left(uv\right)_x=0. \end{aligned} \tag{1.1}$$

The (2+1)-dimensional BKK system is a model that describes wave propagation in nonlinear media with dispersive and dissipative effects [8, 9]. Several attempts have been made by various scholars to investigate the nature of solutions to equation (1.1), to mention but a few; signi_cant exact soliton solutions are obtained by using Backlund transformation and standard truncated Painleve analysis [10]. Direct algebraic method is applied by [11] and new types of complex solutions are obtained. New exact excitations and soliton fission and fusion for Eqn (1.1) have been investigated by an extended mapping approach [12]. Backlund transformation and the Hirota bilinear method have been implemented to investigate the solutions to this equation [13]. In general, many attempts have been made by different researchers to investigate some behaviors of equation (1.1) [14-18].

II. General Facts of SGEM

The sine-Gordon expansion method is based on sine-Gordon equation and travelling wave transformation [19,20]. Lets consider the following equation [21]

$$u_{xx}-u_{tt}=m^2\sin\left(u\right), \tag{2.1}$$

where $u=u\left(x,t\right)$, and m is real constant. When we apply the wave transform $\xi=\mu\left(x-ct\right)$ to Eq.(2.1), we obtain the nonlinear ordinary differential equation (NODE) as following;

$$U''=\frac{m^2}{\mu^2\left(1-c^2\right)}\sin\left(U\right), \tag{2.2}$$

where $U=U\left(\xi\right)$, and, ξ is the amplitude of the travelling wave, c is the velocity of the travelling wave. If we reconsider Eq.(2.2), we can write in the fullsimplified version as following;

$$\left[\left(\frac{U}{2}\right)'\right]^2=\frac{m^2}{\mu^2\left(1-c^2\right)}\sin^2\left(\frac{U}{2}\right)+K, \quad .3)$$

where K is the integration constant. When we resubmit as $K=0,\ w(\xi)=\frac{U}{2},$ and $a^2=\frac{m^2}{\mu^2\left(1-c^2\right)}$ in Eq.(2.3), we can obtain following equation;

$$w'=a\sin\left(w\right). \quad 2.4)$$

If we put as $a=1$ in Eq.(2.4), we can obtain following equation;

$$w'=\sin\left(w\right). \tag{2.5}$$

If we solve Eq.(2.5) by using separation of variables, we find the following two significant equations;

$$\begin{aligned}\sin\left(w\right)=\sin\left(w\left(\xi\right)\right)&=\left.\frac{2pe^{\xi}}{p^2e^{2\xi}+1}\right|_{p=1},\\ &=\sec h\left(\xi\right),\end{aligned} \tag{2.6}$$

or

$$\begin{aligned}\cos\left(w\right)=\cos\left(w\left(\xi\right)\right)&=\left.\frac{p^2e^{2\xi}-1}{p^2e^{2\xi}+1}\right|_{p=1},\\ &=\tanh\left(\xi\right),\end{aligned} \tag{2.7}$$

where p is the integral constant and non-zero. For obtaining the solution of following nonlinear partial differential equation;

$$P\left(u,u_x,u_t,\cdots\right. \tag{2.8}$$

let's consider the travelling wave solution as

$$\begin{aligned}U\left(\xi\right)=\sum_{i=1}^{n}\tanh^{i-1}\left(\xi\right)[B_i\sec h\left(\xi\right)\\ +A_i\tanh\left(\xi\right)]+A_0.\end{aligned} \tag{2.9}$$

We can rewrite Eq.(2.9) according to Eqs.(2,6) and Eq.(2.7) as following;

$$U(w)=\sum_{i=1}^{n}\cos^{i-1}(w)[B_i\sin(w)+A_i\cos(w)]+A_0. \quad (2.10)$$

Under the terms of homogenous balance technique, we can determine the values of n under the terms of $NODE$. Let the coefficients of $\sin^i(w)\cos^j(w)$ all be zero, it yields a system of equations. Solving this system by using Wolfram Mathematica 9 give the values of A_i, B_i, μ, c. Finally, substituting the values of A_i, B_i, μ, c in Eq.(2.10), we can find the new travelling wavesolutions to the Eq.(2.8).

III. Implementation of SGEM

In this section, we consider solving Eqn. (1.1) by using SGEM. Lets consider the the (2+1)-dimensional BKK system given in Eqn. (1.1). Using the transformation $u=U(\xi)$, $\xi=\mu(x-ct)$, Eqn. (1.1) reduces to the following NODE:

$$\mu^2U''-2U^3+3cU^2-c^2U=0, \quad (3.1)$$

under the terms of $v=0.5\left(\mu U'+cU-U^2\right)$.

using the balance principle on the highest derivative U'' and the nonlinear term U^3 in Enq. (3.1), yields n=1. Using n = 1 along with Eqn. (2.10), we have:

$$U(w)=B_1\sin(w)+A_1\cos(w)+A_0, \quad (3.2)$$

$$U''(w)=, \quad (3.3)$$

$$\vdots$$

Substituting Eqns. (3.2) and (3.3) in Eqn. (3.1), we obtain a system including some trigonometric functions. If we solve this system of trigonometric functions by equating all the coeffcients of the trigonometric identities of the same power to zero with help of Wolfram Mathematica 9, we can find some coefficients for B_1, A_1, A_0 and so on . We substitute in each case the obtained results of the coefficients in Eqn. (2.9), to obtain the new travelling solutions to equation (1.1).

Case-1:

$$A_0=\frac{c}{2}, A_1=-\frac{c}{2}, B_1=0, \mu=\frac{c}{2},$$

which gives:

$$u_1(x,\ y,\ t)=\frac{1}{2}\left(c-c\,tanh\left[\frac{1}{2}c(x+y-ct)\right]\right) \quad (3.4)$$

and

$$v_1(x,\ y,\ t)=-\frac{1}{8}c^2sech^2\left[\frac{1}{2}c(x+y-ct)\right] +\frac{1}{4}c\left(c-c\,tanh\left[\frac{1}{2}c(x+y-ct)\right]\right) -\frac{1}{8}\left(c-c\,tanh\left[\frac{1}{2}c(x+y-ct)\right]\right)^2. \quad (3.5)$$

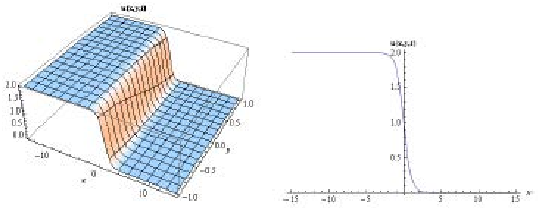

Figure 1: The 3D and 2D surfaces of Eqn. (3.4) by considering the values $c=2$, $t=0.002$, $-15<x<15$, $-1<y<1$ and $y=0.001$ for the 2D graphic.

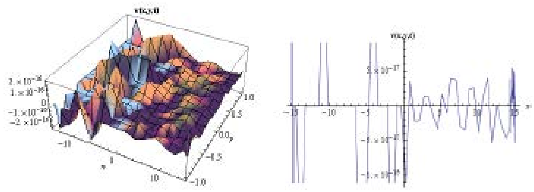

Figure 2: The 3D and 2D surfaces of Eqn. (3.5) by considering the values $c=2$, $t=0.002$, $-15<x<15$, $-1<y<1$ and $y=0.001$ for the 2D graphic.

Case-2:

$$A_0=-\mu, A_1=\mu, B_1=0, c=-2\mu,$$

which gives:

$$u_2(x,\ y,\ t)=\mu\Big(-1+tanh[\mu(x+y+2\mu t)]\Big) \quad (3.6)$$

and

$$v_2(x,\ y,\ t)=\frac{1}{2}\mu^2sech^2[\mu(x+y+2\mu t)] -\mu^2\Big(-1+tanh[\mu(x+y+2\mu t)]\Big) -\frac{1}{2}\mu^2\Big(-1+tanh[\mu(x+y+2\mu t)]\Big)^2. \quad (3.7)$$

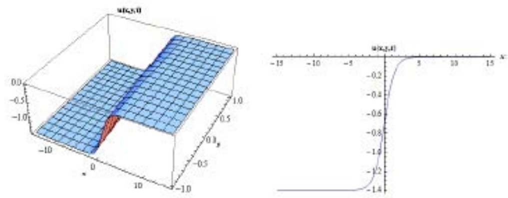

Figure 3: The 3D and 2D surfaces of Eqn. (3.6) by considering the values $\mu=0.7$, $t=0.002$, $-15<x<15$, $-1<y<1$ and $y=0.001$ for the 2D graphic.

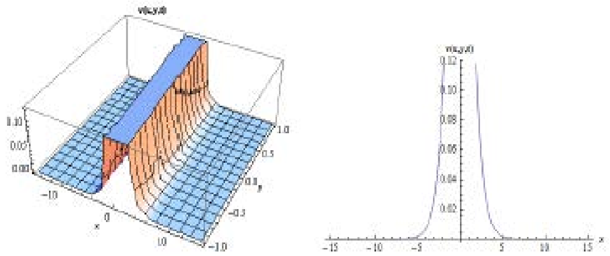

Figure 4: The 3D and 2D surfaces of Eqn. (3.7) by considering the values $\mu = 0.7$, $t = 0.002$, $-15 < x < 15$, $-1 < y < 1$ and $y = 0.001$ for the 2D graphic.

Conclusion

In this paper, we have successfully employed the powerful sine-Gordon expansion method to the (2+1)-dimensional BKK system. We elegantly obtained some new hyperbolic function solutions to equation (1.1). The obtained solutions to this equations in this paper are indeed verified to be the solutions to this equations with help of Wolfram Mathematica 9. Thus, to obtain such solitary wave solutions for the model in wave propagation in nonlinear media with dispersive effect it is very important to know the physical meaning of the model. For instance, the hyperbolic cosine function is the shape of a hanging cable and the hyperbolic tangent arises in the calculation and rapidity of special relativity [22]. We have seen that some results found in this paper are related to special relativity. We observed that the solutions to both equation(1.1) are newly constructed solutions when compared with the solutions obtained by [7]. The results obtained show that sine-Gordon expansion method is an efficient and easy method that can be to applied to various real life models describe by nonlinear partial differential equations.

To the best of our knowledge, the application of SGEM to (1.1) has not been submitted to literature in advance.

References

[1] H.M. Baskonus and H. Bulut, “Exponential prototype structures for (2+1)-dimensional Boiti-Leon-Pempinelli systems in mathematical physics”, Waves in Random and Complex Media, 26(2), 189-196, 2016.

[2] H.M. Baskonus, H. Bulut and F.M. Belgacem, “Analytical Solutions for Nonlinear Long-Short Wave Interaction Systems with Highly Complex Structure”, Journal of Computational and Applied Mathematics, 15 pages, 2016.

[3] M.A. Abdou and A.A. Soliman, “Variational Iteration method for Solving Burger’s and Coupled Buger’s Equations”, Journal Computational and Applied Mathematics . 181, 245-251, 2005.

[4] M.A. Noor, K.I. Noor, A. Waheed and E.A. Al-Said, “Some new solitonary solutions of the modified Benjamin-Bona-Mahony equation”, Computer and Mathematics with Applications. 62, 2126-2131, 2011.

[5] E.M.E. Zayed and K.A.E. Alurrfi, “The Modified Extented Tanh-Function Method and its Applications to the Generalized KdV-mKdV Equation with Any-Order Nonlinear Term”, International Journal of Enviromental Engineering Science and Technology. 1(8), 165-170, 2013.

[6] C. Ye, “New Exact Solutions for the Generalized mKdV Equation with variable coefficients”, Applied Mathathematical Sciences, 5(75), 1484-1490, 2010.

[7] M. Song, S. Li and J. Cao, “New Exact Solutions for the (2+1)-Dimensional Broer-Kaup-Kupershmidt Equations”, Abstract and Applied Analysis, 9 pages, 2010.

[8] M.M. Hossain, H.O. Roshid, M.A.N. Sheikh, “Abundant Exact Traveling Wave Solutions of the (2+1)-Dimensional Couple Broer-Kaup Equations”, Journal for Foundations and Applications of Physics, 3(1), 13 pages, 2016.

[9] E. Yomba, “The Modified Extended Fan Sub-Equation Method and its Application to the (2+1)-Dimensional Broer-Kaup-Kuperschmidt Equations”, Chaos, Solitons and Fractals. 27(1), 187-196, 2006.

[10] Y. Jin-Ping and L. Sen-Yue, “Abundant Coherent Structures of the (2+1)-Dimensional Broer-Kaup-Kupershmidt Equation”, Zeitschrift fur Natur-forschung A. 56, 619-625, 2001.

[11] N. Taghizadeh and A. Neirameh, “New Complex Travelling Wave Solutions to the Nonlinear Broer-Kaup-Kupershmidt System”, Middle-East Journal of Scientific Research. 11(11), 1542-1545, 2012.

[12] F. Jian-Ping and Z. Chun-Long, “New Exact Excitations and Soliton Fission and Fusion for the (2+1)-Dimensional Broer-Kaup-Kupershmidt System”, Chinese Physics B. 14(4), 669-675, 2005.

[13] X.Y. Wen, “N-Soliton Solutions and Localized Structures for the (2+1)-Dimensional Broer-Kaup-Kupershmidt System”, Nonlinear Analysis: RealWorld Applications. 12(6), 3346-3355, 2011.

[14] C.L. Bai and H. Zhao, “A New General Algebraic Method and its Applications to the (2+1)-Dimensional Broer-Kaup-Kupershmidt Equations”, Applied Mathematics and Computation. 217(4), 1719-1732, 2010.

[15] X.R. Hu and Y. Chen, “Nonlocal Symmetries, Consistent Riccati Expansion Integrability, and their Applications of the (2+1)-Dimensional Broer-Kaup-Kupershmidt System”, Chin. Phys. B. 24, 090203 2015.

[16] J. Lin and H.M. Li, "Painleve Integrability and Abundant Localized Structures of (2+1)-Dimensional Higher Order Broer-Kaup System", Z. Naturforsch. 57, 929-936, 2002.
[17] A.G. Davodi, D.D. Ganji and A. Asgari, "Finding General and Explicit Solutions (2+1)-Dimensional Broer-Kaup-Kupershmidt System Nonlinear Equation by Exp-Function Method", Applied Mathematics and Computation. 217(4), 1415-1420, 2010.
[18] C.L. Chen and S.Y. Lou, "CTE Solvability and Exact Solution to the Broer-Kaup System", Chin. Phys. Lett. 30(11), 110202, 2013.
[19] C. Yan, "A Simple Transformation for Nonlinear Waves", Physics Letters A, 22(4), 77-84, 1996.
[20] H.M. Baskonus, "New AcousticWave Behaviors to the Davey-Stewartson Equation with Power Nonlinearity Arising in Fluid Dynamics", Nonlinear Dynamics, 86(1), 177-183, 2016.
[21] Z. Yan and H. Zhang, "New Explicit and exact Travelling Wave Solutions for a System of Variant Boussinesq equations in Mathematical Physics", Physics Letters A, 252, 291-296, 1999.
[22] E.W. Weisstein, "Concise Encyclopedia of Mathematics", 2nd edition. New York: CRC Press, 2002.